What Is Wrong with this Picture? A Problem with Comparative Return Plots on Finance Websites and a Bias Against Income-Generating Assets

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What Is Wrong with this Picture? A Problem with Comparative Return Plots on Finance Websites and a Bias Against Income-Generating Assets

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University of Maine

This paper brings to light and discusses a systemic issue in the calculation and display of relative return information as currently seen on some of the most prominent finance websites; income-generating events such as dividends and interest are not included in relative return calculations and all comparative return graphics. The resulting ranking of the securities, based on such incomplete returns, is essentially meaningless from a total return perspective, yet they are being served to millions of investors every day. This could lead to the formation of a possible availability heuristic and an optical bias against fixed-income and other income generating assets. This problem has gone unnoticed for many years with no discussion of the topic either in the academic or practitioner press. The ready availability of such unclear or inaccurate information from sources generally perceived to be credible can, in this age of do-it-yourself portfolio management, have serious and damaging financial consequences to the unsuspecting investor. The paper also shows the effect of this return differential on the calculation of the asset correlation matrices and the subsequent effect on the resulting asset-weight vectors that are used to generate Markowitz style mean-variance portfolios. The visual discrepancies are then supported by the application of the Gibbons, Ross and Shanken [1989] W-test for portfolio efficiency. The authors' proposed correction, based on elementary finance, fixes the problem.

Keywords: Comparative return charting, Portfolio efficiency, Total returns, Dividends, Finance websites

Investors are concerned with total return. This is a basic tenet of finance. They may differ in their preferences for current income versus capital gains, but total return is what eventually matters. Investors would not ignore a portion of that return, either. So when we analyze the relative performance of stocks, bonds, exchange traded funds (ETFs) or any security over time, it is total return that we wish to examine. Unfortunately, that is not what we see when we utilize the most visible and trusted Web sources that provide information to more than 41 million retail and institutional investors per month.

This paper brings to light and discusses a systemic issue in the calculation and display of relative return information as seen on some of the most prominent finance sites on the Web: income-generating events such as dividends and interest are not included in return calculations and comparative return graphics, thus resulting in the formation of a possible availability heuristic (Shefrin [2005]) and an optical bias against fixed-income and dividend paying securities. We call it the “Compare To” problem.

A deeper problem with the issue we have uncovered is that, despite the structural simplicity and obvious implication of the error, it has gone unnoticed for many years with no discussion of the topic either in the academic or practitioner press, yet its effects could be subliminal and far-reaching. The problem is algorithmically embedded within the feature and can, apart from creating confusion and bias, also lead to incorrect asset rankings and a mis-allocation of funds. As Elton, Gruber and Blake [2001] point out, “All data sets have errors. The types of errors that are most harmful are systematic errors that cause biases” (p. 2416). The downward bias introduced in the returns due to the exclusion of income events makes income generating assets seem
relatively unattractive, especially when compared to assets that derive most of their growth from capital appreciation only. This can have the undesirable effect of altering the final asset mix weightings in a portfolio, even if valid investment principles are present and adhered to. The paper also demonstrates this effect by deploying the classic Markowitz [1956] mean-variance optimization process.

This issue regarding the effect of non-inclusion of income-generating events, in the return calculations and relative return graphics, exists on all the major finance portals that the authors have investigated—Google Finance, Yahoo! Finance, Bloomberg.com, Microsoft’s MSN Money and BigCharts.com. Most of these sites are now virtually household names. BigCharts is a part of the Wall Street Journal’s Digital Network and owned by Dow Jones & Company. Bloomberg is very often a part of the essential toolkit of analytic software for most U.S.-based asset managers.

Following a brief example, we will discuss the importance of looking at total returns, introduce the major online finance sites and illustrate the effects of income exclusion on annualized returns for a set of securities, both with and without income-generating events. We also will examine the effect of this pricing differential on the calculation of the asset correlation matrices and demonstrate the effect this can have on the asset-weight vectors that are used to generate constrained and unconstrained Markowitz style minimum-variance portfolios. The visual discrepancies are then supported by the application of the exact Gibbons, Ross and Shanken [1989] W-test for portfolio efficiency, in the mean-variance space.

**A BRIEF EXAMPLE**

We illustrate the “Compare To” problem with an example and a chart. Figure 1 shows the five-year performance of three assets from August 2003 to August 2008 as seen on the Bloomberg website. SPY is the S&P 500 index ETF and is at the top of the graph. The other two assets are the diversified currency fund ICPHX and the long-term Treasury index fund VBLTX. 

SPY appears to be the obvious and historically successful return generating choice. From the graph, it appears that SPY has a 30% return over this period, while ICPHX has a small positive return (about +5%) and VBLTX has a small net loss (about -1%). Unquestionably, based on this chart generated by Bloomberg, most investors would have preferred to hold the S&P 500 over this period, since the other two assets never seem to be going anywhere; in fact, its recent weakness (as seen on the chart) could be seen by some as an opportunity to increase portfolio exposure toward it. The problem, however, is that the returns and return differentials as implied in Figure 1 are not correct and would have consequently led to suboptimal portfolio allocations and asset selections.

Figure 2 shows the same three securities over the exact same period, but in this case the chart was generated by the authors and based on their correction to the chart-generating algorithm. The correction incorporated income events in the return calculation. The return series is based on adjusted prices that include all income-generating events (e.g., dividends, interest). This chart indicates that the currency fund ICPHX outperformed the S&P 500, returning 41% over the period. The Treasury bond fund VBLTX actually generated 30% over the period, contrary to the Bloomberg chart, which seemed to imply a negative return (all assets have a starting point of 1). Notice that the return on the S&P 500 is higher by about 9% in our corrected chart, which can be attributed to the inclusion of the dividend yield on the S&P 500 return (approximately 1.4% per annum over five years). In this example, the Bloomberg chart did not account for income events; our correction did. This omission causes a serious misrepresentation of asset performance and relative performance orderings, where well-performing assets could be dismissed due to perceived underperformance. Later in the paper, we will show that in a mean-variance optimization framework, the implication of such an inversion in the ranking of relative returns can be severe and lead to misallocation of funds as a result of an incomplete and biased information set available to the investor.

As our example demonstrates, ignoring income events is a significant omission, leading to potentially significant errors in performance assessment and allocation decisions. Because dividend-paying firms are much larger and profitable than non-dividend paying firms (Grullon and Michaely [2002]), a bias against dividends is also a bias against larger firms. It is also a major bias against the entire asset class of fixed income securities.

**DO DIVIDENDS AND INTEREST MATTER?**

Dividends and interest do matter, and sometimes more than other times. Dividends play a signaling role (e.g., Bhat-charya [1979], John and Williams [1985], Miller and Rock [1985]), reduce agency costs (Jensen [1986]) and have tax implications. There is evidence that investors prefer dividends in down markets, apparently desiring the more certain “bird-in-the-hand” of dividends at such times (Fuller and Goldstein [2005]).

What really matters is that dividends deliver a portion, often a large portion, of an investor’s return. Recall the basic formula for return (R) given price (P) and dividend (D):

\[ R_{i,t} = (P_{i,t} - P_{i,t-1} + D_{i,t})/P_{i,t}. \] (1)

Noer [2002] reminds us of a fascinating statistic: “One dollar invested in the S&P 500 in 1926 would be worth around $2,260 now, including reinvested dividends. But take away the dividends and that same dollar would have grown to just $90. Much the same holds true today. Over the last 20 years, dividends have accounted for nearly 50% of the total return.
FIGURE 1 Performance of three assets from Bloomberg.com.
Return comparison generated by Bloomberg.com for SPY (S&P 500 ETF), ICPHX (currency fund, up about 5%), and VBLTX (Treasury Fund, down about –1%), Aug-2003 to Aug-2008. SPY appears to be the clearly dominating return asset (up about 30%).

FIGURE 2 Performance of the same three assets using the authors’ correction.
Return comparison generated by the authors for SPY (S&P 500 ETF), ICPHX (currency fund), and VBLTX (Treasury Fund), Aug-2003 to Aug-2008. This depiction includes income events. Notice that the return disparity has significantly diminished (compared to Figure 1), in fact ICPHX is now the best performing asset, up 41%. Inclusion of income in the relative return calculations has produced an alteration in the performance ranking of the three assets. The Bond fund that appeared to have a –1% return in Figure 1, has an actual return of 30% over the exact same period.
of the S&P 500.” Fuller and Goldstein [2005] found that from 1970 to 2000, dividend-paying stocks outperformed non-dividend paying stocks, the ones we often think of as high-growth stocks. The case for fixed income securities is even simpler; not including the interest payments associated with the security severely reduces the associated total return (see Table 2). Thus we ignore total return at our peril. The exclusion of dividends and interest, in the return-generating mechanism, leads to a downward bias, and the use of such returns to rank securities would be essentially meaningless from a total return perspective. Yet we continue to see relative return graphs all over the Internet that are based on partial total returns.

FINANCE WEBSITES

Individual investors are increasingly turning to the Web to obtain information on the financial markets. In addition, academics and other researchers are increasingly using online resources, as methods are developed to retrieve financial data (e.g., Hasbrouck [2003], Corrado and Miller [2006], Mohamed and Al-Jaroodi [2007]). Google Finance, Yahoo! Finance, Bloomberg, MSN Money and BigCharts are among the top finance websites. All finance portals provide a reasonably rich and sophisticated set of graphing tools to the visitor at virtually no direct cost. They have multiple options where entering a ticker brings up a graphical display of historical performance and optional technical indicators. They also provide earnings estimates, fundamental and even quantitative style data, apart from live news and fee-based research reports. In a way they have done an outstanding job reducing the informational asymmetries between the institutional and retail investor.

These sites have enormous reach. For example, BigCharts is one of the most comprehensive and developed sites that delivers financial analytics over the Web. It is owned by Dow Jones & Company and operates under the Wall Street Journal Digital Network. BigCharts also licenses and provides its output to visible industry members such as MerrillLynch.com, Morgan Stanley, UBS, Citibank, Fidelity.com, WSJ.com, Financial Times, New York Times, and Barron’s, as well as brokers such as Ameritrade.com, Schwab.com and Marketwatch.com. BigCharts’ clients form a financial who’s who list (see Appendix A). BigCharts also licenses its technology and builds customized client solutions. The Wall Street Journal Digital Network claims to service at least 39 million unique visitors per month with more than 616 million monthly page views [http://dowjonesonline.com]. An earlier report by Nielsen/NetRatings released in September 2007 indicates that the above-mentioned sites had an aggregate of about 41 million unique visitors for the month of August 2007 (see Appendix B for individual site statistics). The visibility and reporting burdens that come attached with access to such a massive audience are immense.

SECURITY SELECTION AND STUDY PERIOD

A synopsis of the problem was provided in our earlier example. In this section, we illustrate systematically what is occurring when an investor uses any of these websites. We select a set of eight highly liquid securities; together these securities cover all major asset classes, comprise a highly diversified portfolio and were chosen to have minimal overlap. In addition, these securities are tradeable and have continuous pricing history available since December 31, 1999. This was chosen as the starting point since Google begins all its mutual fund charts only at this point. (Yahoo and BigCharts have the ability to go back earlier, while Bloomberg can only go back five years.) In addition, this period includes at least one full bear market and one full bull market, thus insulating the study from any form of market-phase bias. The securities are a mixture of common stocks, mutual funds and an ETF. This also eliminates any specific security-class bias. We will show that the problem applies to any type of traded security. The eight securities are listed in Table 1. All graphs were drawn on August 7, 2008. The return data period, as a consequence for each of the selected assets was December 31, 1999, to August 7, 2008.

There are two stocks in the study: one pays a steady stream of dividends (Pfizer, PFE) and the other has had no dividends since its inception in 1990 (Genzyme, GENZ). Both are bellwether stocks and have the highest market capitalization in their respective industries (Pharmaceuticals, Biotechnology). Our ETF is the S&P 500 index tracking ETF (SPY), which is the most actively traded security on the U.S. exchanges with an average daily trading volume (three-month moving average) of about 260 million shares; by comparison the trading volume of Pfizer, Genzyme, General Electric and Exxon Mobil are 50 million, 3 million, 63 million and 29 million shares, respectively.

In addition to the S&P 500 ETF, five mutual funds were chosen to create a diversified multiasset class portfolio.

### TABLE 1
Sample Portfolio Assets

<table>
<thead>
<tr>
<th>Asset name</th>
<th>Ticker</th>
<th>Market Cap</th>
<th>Inception Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P500 Depository Receipts</td>
<td>SPY</td>
<td>65B</td>
<td>Jan, 1993</td>
</tr>
<tr>
<td>(Spiders) S&amp;P500 Tracking ETF</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LT Treasury Bond Index - Vanguard</td>
<td>VBLTX</td>
<td>2.8B</td>
<td>Jun, 1996</td>
</tr>
<tr>
<td>Total Int’l Stock Index –Vanguard</td>
<td>VGTSX</td>
<td>27B</td>
<td>Jun, 1996</td>
</tr>
<tr>
<td>Fidelity Select Gold Fund</td>
<td>FSAGX</td>
<td>2.3B</td>
<td>Jan, 1987</td>
</tr>
<tr>
<td>Franklin Templeton Hard Currency Fund</td>
<td>ICPHX</td>
<td>0.69B</td>
<td>Jun, 1995</td>
</tr>
<tr>
<td>REIT Index Fund – Vanguard</td>
<td>VG6X</td>
<td>7.29B</td>
<td>Jun, 1996</td>
</tr>
<tr>
<td>Pfizer Inc.</td>
<td>PFE</td>
<td>130B</td>
<td>Jan, 1982</td>
</tr>
<tr>
<td>Genzyme Corp.</td>
<td>GENZ</td>
<td>20B</td>
<td>Mar, 1990</td>
</tr>
</tbody>
</table>

Table 1 lists the assets that we chose for the study, their tickers, market capitalizations and inception dates. Each security proxies for an asset class and is also a tradable liquid asset.
A PROBLEM WITH COMPARATIVE RETURN PLOTS

Figure 3 is what a user will see on the ‘advanced chart’ option on BigCharts, it includes the % Compare sub-panel which plots the relative return differential between the securities plotted. It appears the VBLTX has generated a cumulative return of about 14% over the 8-1/2 year period. That seems to be very low, considering that VBLTX is a long-term Treasury index fund.

Between the SPY and the VGTsx, the largest and most liquid common stocks in the United States, Europe, Australasian and Emerging markets are represented. The Treasury bond, gold, hard currency and real-estate classes are proxied by VBLTX, FSAGX, ICPHX and VGSIX funds, respectively.5

This set of highly liquid assets is chosen to minimize redundancies, illustrate the effects of income exclusion from price series, yet have a relatively complete coverage of tradeable proxies for the broad asset classes (for mean-variance optimization purposes as shown later in the paper).

GENERATING A COMPARISON GRAPH

In the “Compare To,” “% Compare,” or “Compare” feature, one or more securities can be compared to a base security. All of the online finance sites offer this feature in their charting section. It is a commonly used option for investors who wish to see how their investments are performing relative to a baseline security. Unlike the basic chart, which normally shows a security’s price over time, the Compare chart displays percentage changes, on the Y-axis, over the time period selected.

Appendix C is a direct cutout from the BigCharts website, which explains what the % Compare feature does. Note that the BigCharts explanation claims that the chart shows “relative performance” and that the feature indicates if one company is outperforming or underperforming the stock of another. Outputs from Yahoo! Finance, Google Finance, Microsoft’s MSN Money and Bloomberg are in similar percentage return format and will be discussed shortly. Google even provides a percentage return number over the range of the period for which the chart is drawn. (See Appendix D for instructions for generating the compare graph for a selection of sites.)

Figure 3 is what a user will see if they choose BigCharts then click on “advanced chart,” enter SPY followed by VBLTX, and choose % Compare on the lower indicator. A curious user may spend a few minutes to discern what % Compare is telling them (the lower panel in the chart). It appears that since the year 2000 (December 31, 1999, to be precise), which is entered in the Custom timeframe box, the VBLTX (Vanguard Long-term Treasury index) has outperformed the S&P 500 by about +25%. Moreover, it appears that VBLTX has generated a return of about 14% over the 8-1/2 year period (upper chart), which is about 1.6% per year. This second piece of information could raise a question in the mind of a user who is conversant with historical attributes of various asset classes. The Ibbotson Report (SBBI [2006]) lists the average annual return on U.S. long-term Treasury bonds to be about 5.9%. Clearly something...
is missing, but there is little to indicate the omission and the cause for the discrepancy in the annual bond returns.

As we have suggested, this issue is not limited to the Bloomberg or the BigCharts websites. An examination of three other major finance portals reveals the persistence of this omission. The information is the same at Microsoft’s MSN Money (Figure 4). To conserve space we have included PFE as well as a comparison with the SPY, along with VBLTX in the same graph. It may be a little harder to estimate the numerical numbers associated with the graphical output but the VBLTX line is relatively flat at about 14%, SPY around −14%, and PFE at a −40% return over the exact same period as the output from BigCharts above. Note that while Pfizer did not do well in this period; its return when including income events (dividends) was not quite as bad as it appears (−3.3% per annum in reality versus the −5.5% per annum, Table 2).

To complete the comparison, we also produced graphs of SPY, VBLTX and PFE from the Yahoo! Finance website (Figure 5) and the Google Finance site (Figure 6). In both cases, the information and its presentation are similar. The Compare graph from Google does appear to have one minor advantage over the other sites; that is, it provides specific numerical values of the percent returns associated with each security in the graph. For SPY, VBLTX and PFE, the numbers are −13.28%, +14.55% and −43.71%, respectively. Thus the user does not have to guesstimate the numerical return values from the location of the lines on the graphical output. There is, however, no indication to the viewer that such comparative return information for Google does not include any income event whatsoever, and thus produces a downward bias in the cumulative return numbers or the graphical ordering of the securities being researched. Note that none of the sites includes an option for including income events in the return comparison.

### ANNUALIZED RETURNS WITH AND WITHOUT INCOME

The annualized returns associated with each of the selected assets over the same time period (December 31, 1999–August 7, 2008, as used in the graphs above) are presented in Table 2. The non-inclusion of dividends or interest leads to a different set of annualized returns. The effect is more pronounced for bonds, gold stocks, currency, real estate investment trusts (REITs)—assets that traditionally generate higher income streams. The gold fund is underestimated by 7.1%, long-term government bonds by 5.9%, REITs by 5.7%, and currency by 5.3%; recall these are annualized or per year errors. The compounded effect over time is extraordinary, to say the least. The exclusion of such large annual returns from a comparative return analysis portrays a different image of the asset, from what truly is the case. The total return differential can be thought of as an annual return loss that is not available to the viewer and thus not part of the investor information set. The stock GENZ has a zero return differential because it is a non-dividend paying stock. This security also illustrates the only scenario when the return comparison charts, in their current form, would be accurate. A suboptimal portfolio allocation decision can manifest as a result of the “availability heuristic” that the user would be automatically subject to; that is, agents tend to overweight information that is readily available and intuitive, relative to information that is less salient and more abstract, thus biasing judgments (Shefrin [2005]). The omission of such key information in what is made available to the users of these websites has the potential to affect their portfolio selections adversely.

### CORRELATIONS

Except for BigCharts, the other three sites—Google, MSN Money, and Yahoo! Finance—allow the user to download price data for a particular security and a specified date range. Google’s price history does not even include any dividend payouts, while MSN Money and Yahoo! Finance provide that information in the download. In fact, Yahoo! Finance provides an additional column of pricing data called the “adjusted close” that incorporates dividends, splits, stock dividends and similar corporate actions. The free availability of such pricing data on the Web has drawn users to programmatically download such information and compute their own devices for portfolio analytics (Aggrawal...

<table>
<thead>
<tr>
<th>Annualized Returns (12/31/1999 to 8/7/2008)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P500</td>
</tr>
<tr>
<td>SPY</td>
</tr>
<tr>
<td>Without Income Events</td>
</tr>
<tr>
<td>With Income Events</td>
</tr>
<tr>
<td>Return Differential</td>
</tr>
</tbody>
</table>

Table 2 lists the annualized returns for the set of securities listed in Table 1. The implication of not including income events is best seen in the last row of the table. Genzyme is the exception—because it is the only non-dividend paying security in this set of assets.
A PROBLEM WITH COMPARATIVE RETURN PLOTS

FIGURE 4  MSN Money site comparison chart (12/1999 to 8/2008). Figure 4 shows that the relative return plotting problem persists for a different website – in this case Microsoft’s MSN Money. We also include Pfizer, a dividend paying stock in this chart. This relative return plot excludes income over the entire period for which the three securities are plotted.

FIGURE 5  Yahoo! Finance site comparison chart (12/1999 to 8/2008). Figure 5 shows that the relative return plotting problem persists for yet another website – in this case the heavily visited Yahoo! Finance site. The same three securities are plotted here. This relative return plot also excludes income over the entire period for which the three securities are plotted. This graph looks very similar to Figure 4.

FIGURE 6  Google Finance site comparison chart (12/1999 to 8/2008). Figure 6 shows that the relative return-plotting problem persists for the Google Finance site as well. The same three securities are plotted here. This relative return plot also excludes income over the entire period for which the three securities are plotted, thus significantly understating the returns attributable to the Treasury bond index fund. Figures 4, 5 and 6 are very similar and neither one of them includes income events in the return graphics.
Table 3 has two panels that show the correlation matrices for the without and with income return series. Changes to the correlations are modest. VBLTX and ICPHX correlations are the most affected since they have larger income events embedded in their total returns.

Changes to the correlations (when we add income events to the returns) are modest but not without importance. The ones that change the most are those for bonds and currency (see the columns for VBLTX and ICPHX). It is expected that if these two assets are included in a portfolio then the optimal weights for the assets will vary and be dependent on which of the two correlation matrices is utilized for the mean-variance optimization.6

The estimation of the covariance between two assets is ultimately affected, since \( \text{cov}(x, y) = \sigma_x \sigma_y \rho_{xy} \), where \( \sigma \) is standard deviation and \( \rho \) is the correlation, which ultimately affects the level of systemic risk (\( \beta \)) and portfolio allocations (asset weights).

Changes to the correlations (when we add income events to the returns) are modest but not without importance. The ones that change the most are those for bonds and currency (see the columns for VBLTX and ICPHX). It is expected that if these two assets are included in a portfolio then the optimal weights for the assets will vary and be dependent on which of the two correlation matrices is utilized for the mean-variance optimization.6

\[ \text{Correlation} (x, y) = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sqrt{\sum (x - \bar{x})^2 \sum (y - \bar{y})^2}} \]  

\[ (2) \]
A PROBLEM WITH COMPARATIVE RETURN PLOTS

TABLE 4
Minimum Variance Portfolios (12/1999 to 8/2008)

<table>
<thead>
<tr>
<th>Portfolio Weights</th>
<th>Portfolio St Dev</th>
<th>Portfolio Return</th>
<th>Return/ Risk</th>
<th>SPY</th>
<th>VBLTX</th>
<th>VGTSX</th>
<th>FSAGX</th>
<th>ICPHX</th>
<th>VGSIX</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A: Returns with and without income</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>With returns excluding Income</td>
<td>7.07</td>
<td>3.13</td>
<td>0.44</td>
<td>16.5</td>
<td>29</td>
<td>7</td>
<td>7</td>
<td>29</td>
<td>11.5</td>
</tr>
<tr>
<td>With returns including Income</td>
<td>6.99</td>
<td>7.63</td>
<td>1.09</td>
<td>16.3</td>
<td>29</td>
<td>7</td>
<td>7</td>
<td>29</td>
<td>11.7</td>
</tr>
<tr>
<td>Panel B: Both returns including Income</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weights calculated using wrong data</td>
<td>6.99</td>
<td>7.61</td>
<td>1.09</td>
<td>16.5</td>
<td>29</td>
<td>7</td>
<td>7</td>
<td>29</td>
<td>11.5</td>
</tr>
<tr>
<td>Weights calculated using right data</td>
<td>6.99</td>
<td>7.63</td>
<td>1.09</td>
<td>16.3</td>
<td>29</td>
<td>7</td>
<td>7</td>
<td>29</td>
<td>11.7</td>
</tr>
</tbody>
</table>

Table 4 lists the optimal weights associated with the minimum variance portfolio on the efficient frontier. The minimum variance portfolio is also the left-most point on the curves in Figure 7. The optimization is run on two separate return series, one that does not include income events and the other that does. As seen in Panel A, the standard deviation is lower by about 0.08% for the portfolio that includes income events, while the return is higher by 4.5%, in absolute terms. Notice that the portfolio weights are almost identical in the two cases (bounded by a min of 7% and a max of 29% to prevent corner solutions).

OPTIMAL ASSET ALLOCATION ERRORS

In this section we use actual data (returns for each of the eight assets are over the period December 31, 1999–August 7, 2008) to simulate the various mean-variance frontier portfolios that can be created by using two separate return series on the same set of assets. The series differ in their treatment of income events, thus for each asset we have two return vectors. We will assume the investor is interested in creating efficient portfolios that utilize the Markowitz mean-variance optimization process. The mean-variance efficient portfolio selection problem is one where the investor seeks to minimize the portfolio variance subject to the budget and target return constraint. A short selling non-negativity constraint is optional, depending on the model. In this exercise the non-negativity constraint is enforced. Simply stated, the problem is to:

\[
\text{Minimize } \sigma^2(x) = x^T \Sigma x
\]
subject to,

\[
x^T e = 1 \text{ where } e^T = [1, 1, \ldots, 1] \quad (3)
\]
\[
x^T \mu = \mu_p
\]
and \(x \geq 0\) (optional)

where

\(\mu\) and \(x\) are n-vectors composed of asset rates of return and portfolio weights respectively;

\(\Sigma\) is an \(n \times n\) positive-definite non-singular covariance matrix (the positive definiteness of \(\Sigma\) ensures that the value of the quadratic norm \(\sigma^2(x)\) will be positive for all \(x > 0\), essentially ensuring a positive variance (Greene [1993]);

\(e\) is a unit vector; and

\(\mu_p\) is a scalar equal to the targeted portfolio return.

Excluding the two stocks in our asset set, we have representation of all the primary asset classes—U.S. equities (SPY), international equities (VGTSX), U.S. Treasuries (VBLTX), gold (FSAGX), real estate (VGSIX) and hard currency (ICPHX). An Ibbotson-NAREIT [2006] report shows

Table 5 displays information pertaining to the points on the frontier that have the maximum feasible return/risk ratio. These points of tangency can be identified by the large dots on the two efficient frontiers as shown separately in Figure 7. In Panel A one can see that the return of the income generating portfolio is over 40% above that of the income excluding portfolio (9.37% versus 6.61%). The return/risk ratio jumps over 90% from 0.65 to 1.24, for the portfolio whose weights are determined using the correct set of returns. Notice that the portfolio calculated using the wrong (no income event) data has a lower return/risk ratio. The portfolio weights have changed rather significantly as well, with a sharp reduction to FSAGX (gold) and an increase to ICPHX (currency). The distortion of returns created by omitting income events has real portfolio allocation and performance effects.

TABLE 5
Maximum Return/Risk Portfolios

<table>
<thead>
<tr>
<th>Portfolio Weights</th>
<th>Portfolio St Dev</th>
<th>Portfolio Return</th>
<th>Return/ Risk</th>
<th>SPY</th>
<th>VBLTX</th>
<th>VGTSX</th>
<th>FSAGX</th>
<th>ICPHX</th>
<th>VGSIX</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>With returns excluding income</td>
<td>10.17</td>
<td>6.61</td>
<td>0.65</td>
<td>7</td>
<td>29</td>
<td>20.2</td>
<td>7.8</td>
<td>29</td>
<td></td>
</tr>
<tr>
<td>With returns including income</td>
<td>7.54</td>
<td>9.37</td>
<td>1.24</td>
<td>7</td>
<td>29</td>
<td>7.7</td>
<td>24.8</td>
<td>24.5</td>
<td></td>
</tr>
<tr>
<td>Panel B: Both returns including Income</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weights calculated using wrong data</td>
<td>10.05</td>
<td>11.58</td>
<td>1.15</td>
<td>7</td>
<td>29</td>
<td>20.2</td>
<td>7.8</td>
<td>29</td>
<td></td>
</tr>
<tr>
<td>Weights calculated using right data</td>
<td>7.54</td>
<td>9.37</td>
<td>1.24</td>
<td>7</td>
<td>29</td>
<td>7.7</td>
<td>24.8</td>
<td>24.5</td>
<td></td>
</tr>
</tbody>
</table>

Table 5 displays information pertaining to the points on the frontier that have the maximum feasible return/risk ratio. These points of tangency can be identified by the large dots on the two efficient frontiers as shown separately in Figure 7. In Panel A one can see that the return of the income generating portfolio is over 40% above that of the income excluding portfolio (9.37% versus 6.61%). The return/risk ratio jumps over 90% from 0.65 to 1.24, for the portfolio whose weights are determined using the correct set of returns. Notice that the portfolio calculated using the wrong (no income event) data has a lower return/risk ratio. The portfolio weights have changed rather significantly as well, with a sharp reduction to FSAGX (gold) and an increase to ICPHX (currency). The distortion of returns created by omitting income events has real portfolio allocation and performance effects.
FIGURE 7  Mean Variance Efficient Frontiers
In Figure 7 the actual efficient frontiers are traced out using the Markowitz optimization process, for the ‘with and without’ income return series. The large solid dots show the maximum return/risk tangency points, while the solid triangular point is based on the ‘incorrect’ return series. Please refer to the section on ‘Optimal Asset Allocation Errors’ for further discussion.

If we create a portfolio using data that does not include income events (inaccurate data) and compare it to a portfolio that uses data that does include income events (accurate data), then the portfolio that includes the extra income events will be superior, at all risk levels. That is what we see in panel A, Table 4. The standard deviation is lower by about 0.08% for the portfolio that includes income events, while the return is higher by 4.5%, in absolute terms. Notice that the portfolio weights are almost identical in the two cases. The 4.5% loss can be attributed to a composite of the yield on these assets that is not included in the price-return calculations. That is also the information loss to the user who views the relative return plots on each of the websites.

Panel A, Table 4, however, shows inaccurate returns. This is because whether or not the chart shows the income events, the holder of the security will receive the income stream. Panel B, Table 4, makes a more interesting comparison. In this panel we compare actual returns (that include income events) of a portfolio based on the optimization of the wrong
returns, and compare it to a portfolio based on the optimization of the correct returns. For the minimum-variance case, the effects are minor, although it does decrease the allocation to SPY and increase VGSIX (real estate). The return to risk ratio is slightly improved if the correct data are used. We repeated the exercise, but now we created portfolios to maximize the return/risk ratio \( \mu/\sigma \). This is where considerable migration of weights and change in portfolio returns is observed (see Table 5). The \((\mu, \sigma)\) parameters of the optimal portfolios are higher than what was attained in the minimum variance case. In Panel A, Table 5, one can see that the return of the income generating portfolio is more than 40% above that of the income excluding portfolio (9.37% versus 6.61%). This difference in returns is not due to better security performance; rather, it is simply due to the exclusion of income events in the underperforming portfolio and the resulting shifts in the portfolio weights. The return/risk ratio jumps over 90% from 0.65 to 1.24 for the portfolio.

FIGURE 8 A comparison of the actual (as seen on the Internet) and corrected relative performance plots for VBLTX (bonds, top line in both) and SPY (equity, bottom line in both; 12/1999 to 8/2008).

The two graph panels show relative performance (the ‘compare’ feature) for the same two securities (SPY, VBLTX), but they differ in the treatment of the income generating events for the calculation of the returns and the return paths. The top graph does not include income-generating events while the bottom graph does. The bottom graph shows accurate relative returns - notice the wider separation. The bottom panel depiction is not available on any finance website and is generated by the authors’ software that makes the income adjustment to returns. Contrary to the top panel, where long-term treasury bonds fund appear to be going nowhere, the bottom panel actually shows that the fund significantly outperformed equities and had a cumulative total return of about 85% for the 81/2 year period.
whose weights are determined using the correct set of returns. Once again, Panel B is the interesting panel. Notice that the portfolio calculated using the wrong (no income event) data has a lower return/risk ratio. The portfolio weights have changed rather significantly, with a sharp reduction to FSAGX (gold) and an increase to ICPHX (currency). The allocation to VGSIX (real estate) has also declined. The risk associated with the suboptimal portfolio is about 10%, which is about 33% higher on a relative basis compared to the optimal portfolio that has a risk of about 7.5%. Clearly, the distortion of returns created by omitting income events can have real portfolio allocation and performance effects. In addition, the impact of this omission leads to a reduced capital allocation toward asset classes that derive a significant portion of their total return from income events. In the simulation shown above, these asset classes are gold, currency, and real-estate.

Recall that the large (red/solid) dot in Figure 7 is the location of the maximum return/risk (tangency) portfolio for each case (Panel A, Table 5, (μ, σ) values). It can be safely said that the “with income” based frontier dominates the other—at each level of risk, the income including frontier has a superior level of return compared to the no-income frontier. Also notice the point indicated with the (green/solid) triangle (10.05, 11.58). This is the maximum return/risk portfolio from Table 5, Panel B, the portfolio whose weights were calculated using the wrong data (no income events included) but whose performance is based on actual returns. The point to be made is that using the wrong data, as provided on these finance portals, leads to a portfolio with a lower return/risk ratio. The visual determination (Figure 7) is confirmed by applying the exact Gibbons, Ross and Shanken [1989] W-statistic, given as:

$$W = \frac{\left( \frac{1 + \theta^2}{1 + \theta^2} \right)^{\frac{1}{2}} - 1}{\psi^2 - 1}$$

With a p-value of $6.27 \times 10^{-49}$, the null hypothesis of portfolio efficiency is easily rejected (see Appendix E for details). All points on the line connecting the risk-free return (1.72% on the Y axis) and the portfolio shown by the triangular point (the optimal range of possible portfolios that mix the risk-free asset and the risky portfolio) lie below the possible combinations of the risk-free asset and the portfolio calculated using accurate income inclusive returns (point 7.54, 9.37).

With the way the comparative returns of two securities are displayed on some of the leading finance portals, the users are most likely unknowingly operating off the efficient frontier that lies within the attainable higher frontier. That will likely lead to a suboptimal allocation of capital toward income-generating asset classes.

**CONCLUSION**

This paper has demonstrated and brought to fore a long-standing and widespread distortion in the depiction of relative return performance of financial securities on all of the major finance Web portals; somehow this has evaded the scrutiny of millions of visitor investors. As shown earlier, the omission by the portals results in significant calculation, ranking, and portfolio allocation errors; the distortion will occur with all income-generating securities. This error is served up to a very large captive audience, and it is possible that their assessment of desirable securities is altered by this misinformation.

To reinforce the message, in Figure 8 we replicate the distortion found on the websites using our template. These charts replicate/reverse-engineer the ‘compare’ feature in which the S&P 500 ETF (SPY) is the baseline, pegged to 1.00 (Y axis) at the beginning of the period. Thus the performance of VBLTX (bonds) is relative to the S&P 500. The two graph panels show the same securities but they differ in the treatment of the income generating events for the calculation of the returns and the price paths. In the first one it appears that VBLTX (long-term Treasury bonds) have outperformed the S&P 500 by 28% over a nine-year period. This is the same view an internet user would find on Bloomberg, Yahoo! Finance, BigCharts, MSN Money or Google. In the bottom graph (of Figure 8) we find that the actual outperformance was significantly higher, at 89%. The comparative plots as seen on the internet underestimate the true performance of income-generating assets.

The result of these distortions is that assets that have a higher income-generating component in their total returns would appear to be less desirable holdings in a portfolio allocation scheme if such comparative graphing is utilized. Such security types include fixed income debt securities and dividend paying equity securities (at the stock, mutual fund, index, or ETF level). If retail investors or financial advisors use the “Compare To” option to estimate the relative attractiveness of two securities, it is possible that they would surmise that the high income-generating security is unattractive and as a result be under allocated to them. The financial crisis of 2008 has led many investors to look into and review their portfolio holdings and allocations, and anecdotal evidence indicates that a large majority were underexposed to assets that generated income or dividends. It would be a stretch to even suggest that the compare feature created such massive losses. It is possible, however, that such graphing features have subliminally and continually reinforced a perception, in the mind of some investors, that income securities have been perpetual under performers, when in fact they were delivering steady returns (as shown in Table 2) and thus be one of the many contributory factors.

It does not have to be so. Most of the sites have dividends or interest income available in their historical price downloads (Google, however, offers only price information,
without consideration for dividends). All that these sites have
to do is include the income information while generating
comparison graphs, by making a few alterations to their graph
generating algorithms. That would give a more complete and
accurate picture of the relative return differentials over time
for the securities being compared and make available a better
information set to the investor. If for some reason it is not
possible for the providers to alter their comparative return
generating algorithms, then the user has the right to see a
disclaimer next to the chart, such as: “These return compar-
ison graphs do not include income events, which may result in
a downward bias in the displayed end-period returns. The im-
lication of this for ranking securities and allocation of funds
should be considered carefully by the user, before basing any
decisions on the displayed chart.”

Ultimately these brand-name finance portals have a re-
 sponsibility to their visitors and should consider corrective
action. What if an investor was making asset selections even
partially influenced by a visual recall of the comparison
charts? The problem becomes very real then. The ready avail-
plication of this for ranking securities and allocation of funds
for the securities being compared and make available a better
information set to the investor. If for some reason it is not
possible for the providers to alter their comparative return
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Ultimately these brand-name finance portals have a re-
 sponsibility to their visitors and should consider corrective
action. What if an investor was making asset selections even
partially influenced by a visual recall of the comparison
charts? The problem becomes very real then. The ready avail-
plicity of such unclear or inaccurate information from sources
generally perceived to be very credible can in this age of
do-it-yourself portfolio management have serious and dam-
aging financial consequences to the unsuspecting investor.
This problem can, however, be easily rectified.

NOTES

1. About 41 million unique visitors per month for
   all of these websites: Google Finance, Yahoo! Fi-
   nance, Bloomberg.com, Microsoft’s MSN Money and
   BigCharts.com (Nielsen/NetRatings [2007]).

2. At this point, the reader is encouraged to verify this by
going to the Bloomberg.com site, entering VBLTX in
the quote box, click on Chart, then enter SPY in the
“Add Security” Box, click on Draw and finally click on
“5y” for a five-year comparative graph. The numbers
will be different from those shown in Chart 1 (from
August 2008), but the effect will be similar. This holds
for other asset selections as well, as long as at least one
of the assets is rich in income events.

3. Numerical data was obtained from the Yahoo! Finance
website (secondary provider), which provides price
history along with splits and dividend information, as
well as adjusted close prices that incorporate corporate
action and income events, for stocks, ETFs and mutual
funds. The primary source of the pricing data is Com-
modity Systems, Inc. (CSI), the suppliers to Google
Finance, Yahoo! Finance and MSN Money. Historical
pricing data for BigCharts.com is provided by Interac-
tive Data Corp, a publicly traded company.

4. Although not common, there are occasional studies on
data reliability. For example, Elton, Gruber and Blake
[2001] examine the bias resulting from omitted mutual

fund data in the CRSP database. Ince and Porter [2004]

examine equity return data from Thomson Datastream
(TDS).

5. These selections are tradeable diversified funds that
also proxy the broader asset classes, and can be the
constituents of a typical portfolio for a visitor to these
sites. They are also highly capitalized and have market
history going beyond 1999.

6. In the market decline since October 2007, there have
been numerous instances where market pundit have
 been lamenting the limited exposure to these two asset
classes in client portfolios.

7. This can be relaxed without loss of generality, but since
mutual funds in the portfolio typically do not allow for
short selling, we apply the constraint.

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APPENDIX A

BigCharts Licensing Partners

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Our Values
About Us
Clients & Partners Technical Support Contact Us

MarketWatch works with over 250 partners, including leaders in the banking, publishing and financial services industries. These partners rely on us to design, develop and deploy flexible, reliable solutions for their websites.

Take a quick look at a few of these partners, and the world class, highly customized content and tools MarketWatch delivers to their sites.

<table>
<thead>
<tr>
<th>Financial Services</th>
<th>Media/Publishing</th>
</tr>
</thead>
<tbody>
<tr>
<td>TD Waterhouse</td>
<td>USA Today</td>
</tr>
<tr>
<td>Bank of America</td>
<td>FT.com</td>
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<tr>
<td>Merrill Lynch</td>
<td>Earthlink</td>
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<td>UBS</td>
<td>The Motley Fool</td>
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<tr>
<td>Charles Schwab</td>
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<td>Tribune Interactive</td>
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<td>Sina Net</td>
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<td>Northern Investments</td>
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APPENDIX B

Business and Financial Website Numbers

Nielsen/NetRatings has released its numbers for the major US financial websites.

Yahoo! (YHOO) Finance remains in first place with over 16.8 million unique visitors in August 2007. Time per person spent on the site is over 23 minutes, also the highest among the top 20 financial destination. In time spent, Wall Street Digital is second at over 22 minutes.

### Top Online Financial News and Information Destinations for August 2007

<table>
<thead>
<tr>
<th>Brand or Channel</th>
<th>Unique Audience (000)</th>
<th>Time Per Person (hh:mm:ss)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yahoo! Finance</td>
<td>16,844</td>
<td>0:23:35</td>
</tr>
<tr>
<td>MSN Money</td>
<td>12,297</td>
<td>0:19:13</td>
</tr>
<tr>
<td>AOL Money &amp; Finance</td>
<td>10,077</td>
<td>0:16:58</td>
</tr>
<tr>
<td>Forbes.com</td>
<td>9,136</td>
<td>0:05:18</td>
</tr>
<tr>
<td>Wall Street Journal Digital</td>
<td>8,445</td>
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</tr>
<tr>
<td>CNNMoney</td>
<td>8,105</td>
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<tr>
<td>Reuters</td>
<td>6,355</td>
<td>0:05:25</td>
</tr>
<tr>
<td>Bankrate.com</td>
<td>3,977</td>
<td>0:07:20</td>
</tr>
<tr>
<td>Bloomberg.com</td>
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</tr>
<tr>
<td>TheStreet.com</td>
<td>3,491</td>
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<td>Motley Fool</td>
<td>3,369</td>
<td>0:16:32</td>
</tr>
<tr>
<td>American City Business Journals Network</td>
<td>2,821</td>
<td>0:03:22</td>
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<td>BusinessWeek Online</td>
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<td>FreeCreditReport.com</td>
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<td>About.com Business &amp; Finance</td>
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<td>Smartmoney</td>
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<td>FT.com</td>
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<td>Google Finance</td>
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<td>0:16:08</td>
</tr>
<tr>
<td>Morningstar</td>
<td>1,466</td>
<td>0:19:24</td>
</tr>
</tbody>
</table>


APPENDIX C

The text below is a direct quotation from BigCharts.com. Notice the italicized text.

**BigCharts %Compare Feature**

**% Compare**

The % Compare indicator shows the relative performance of symbols that you compare to your chart’s focus symbol. It creates a baseline based on your chart’s focus symbol, around which all other stocks, mutual funds or indexes are compared.

For example, if you apply IBM as your focus symbol, and then in the “compare to” section add MSFT, the percent compare indicator will display IBM as a flat line in the middle of the indicator window with MSFT’s performance plotted relative to IBM. If MSFT’s line rises above IBM, it means that MSFT is outperforming IBM. If it declines below IBM, it means that MSFT is underperforming IBM.

Note: this indicator is useful because it returns the final percentage by which the compared symbols underperformed or outperformed the focus symbol.

APPENDIX D

Using the “Compare” Feature

**BigCharts**

Go to BigCharts.com (actually you will go to http://BigCharts.marketwatch.com/). Enter a ticker symbol and choose “advanced Chart.” On left choose time period and click on “compare to” and enter a ticker of choose an index. Click “draw chart.” If you wish to see “% compare” make that choice in “lower indicator.”

**Microsoft’s MSN Money**

Go to http://moneycentral.msn.com/investor/home.aspx. Enter a symbol and click “get quote.” Choose “charts” below the chart showing. Here choose time period and enter symbol(s) to compare to. Click “redraw chart.”

**Yahoo! Finance**

Go to http://finance.yahoo.com/. Enter symbol and click “get quote.” Beneath the small chart choose a time period. When the new chart appears, click “compare” and enter symbol or choose an index.

**Google Finance**

Go to http://finance.google.com/finance. Enter symbol and click “get quote.” The “compare” chart should be showing. Add a symbol or choose an index to compare to.

**Bloomberg**

Go to Bloomberg.com. Enter quote. Choose “Chart” from tab. Enter additional symbols in “add security.” Click “Go.”

APPENDIX E

The GRS Statistic: Geometrical Test for Portfolio Efficiency

Gibbons, Ross and Shanken [1989] devised an exact form statistic to test for the MV efficiency of a given portfolio...
based on its geometric properties. The test is widely used in studies addressing the issue of portfolio efficiency and CAPM deviations (Brennan, Wang, and Xia [2004], MacKinlay [1995], Roll and Ross [1994], Zhou [1993] and Fama and French [1993, 1996]).

The GRS statistic measures the distance, in mean-standard deviation space, between a test portfolio (market index) and a tangency portfolio (on the efficient frontier) and returns a value, which is then used to assess the relative efficiency of the portfolio under consideration. The GRS statistic denoted by \( W \) is given as:

\[
W = \left( \frac{1 + \theta^2_*}{1 + \theta^2_p} \right)^2 - 1 \equiv \psi^2 - 1 \quad (5)
\]

where, \( \theta^* \) is the Sharpe measure of the *ex post* efficient portfolio (ratio of expected excess return to the standard deviation of the excess return), and \( \theta_p \) is the Sharpe measure of the test portfolio. Essentially is a slope measure \(( \bar{r} / \sigma )\) with excess return \(( \bar{r} )\) and standard deviation of return \(( \sigma )\), and is the ray emanating from the origin on the Y-axis connecting to a portfolio in the first quadrant. Note that cannot be less than one since \( \theta^* \) is the slope of the *ex post* frontier and is based on all the assets in the test (including portfolio p).

To accept the efficiency of the test portfolio, \( \psi^2 \) should be close to 1. Larger values of \( \psi^2 \) imply portfolio inefficiency arising out of the increased distance between the test portfolio and the global MV efficient portfolio on the frontier \(( W = -\psi^2 - 1 \to 0 \) implies efficiency). In other words, for values of \( W \) close to zero, the test portfolio cannot be called inefficient.

The test statistic is determined as:

\[
[T(T - N - 1)/N(T - 2)]^* \left[ \frac{1 + \theta^2_*}{1 + \theta^2_p} \right]^2 \equiv X_F \quad (6)
\]

It follows a F-distribution \( \sim \mathcal{F}(N, T-N-1) \), where \( N \) is the number of assets and \( T \) is the number of time series observations on the underlying asset returns.

**H\(_0\):** Portfolio is efficient

The decision rule to reject \( H_0 \) is: \( \text{Rej. } H_0, \text{iff. } F(X_F, N, T-N-1) < \text{a threshold } p\)-value

For the two portfolios discussed in Panel B of Table 5 the various parameters required to determine the GRS statistic can be see in the table below:

<table>
<thead>
<tr>
<th>Tangency Portfolio (*)</th>
<th>Test Portfolio (p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean, ( r )</td>
<td>9.37</td>
</tr>
<tr>
<td>rf</td>
<td>1.72</td>
</tr>
<tr>
<td>sigma, ( \sigma )</td>
<td>7.54</td>
</tr>
<tr>
<td>( \theta = (r-rf)/\sigma )</td>
<td>1.014</td>
</tr>
<tr>
<td>GRS-W</td>
<td>0.033</td>
</tr>
<tr>
<td>N</td>
<td>6</td>
</tr>
<tr>
<td>T</td>
<td>448</td>
</tr>
<tr>
<td>( X_F )</td>
<td>2.455</td>
</tr>
<tr>
<td>p-value</td>
<td>6.27E-49</td>
</tr>
</tbody>
</table>

The GRS test confirms that the test portfolio is not efficient, relative to the tangency portfolio.