The Stock–REIT Relationship and Optimal Asset Allocations

Executive Summary. *In this paper, the marginal effects of changes (due to non-stationarity or estimation errors) in the REIT-stock risk premium and the REIT-stock correlation on the optimal portfolio asset mix of REITs, stocks, and bonds are determined. Employing a mean-variance utility function and considering different levels of investor risk aversion, the findings reveal that the expected return of REITs, relative to that of stocks, is a much more important factor than the REIT-stock correlation in making portfolio decisions. A 1% change in the forecast return for REITs dramatically impacts optimal portfolio allocations for investors of all risk levels. A significant change of 0.1 in the REIT-stock correlation, on the other hand, has only minimal impact on optimal portfolio weights.*

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Mean-variance optimization and analysis is a widely employed procedure that allows portfolio managers to construct optimal portfolio combinations using various asset classes. Optimal portfolios allow investors to maximize expected return for given risk levels or to minimize risk for given expected return levels. The output from mean-variance analysis, however, is only as good as the inputs that go into the model. Traditionally, the historical returns, and the variance-covariance matrix (which employs standard deviations and correlations) of various asset classes such as real estate investment trusts (REITs), stocks, and bonds are employed as a stationary proxy of expected values. The usefulness of this historical data has recently been questioned on several fronts. There is considerable debate and discussion that the returns from equity going forward will not match historical levels. The correlations of the returns of asset classes have likewise been shown to vary considerably over time.

This work makes no attempt whatsoever to predict asset returns or correlations or to construct a stationary variance-covariance matrix. Instead, the paper focuses on mathematically determining the impact of non-stationary estimates (or errors in estimation) of these factors, on the optimal portfolio asset mix (REITs, stocks, and bonds). A widely used mean-variance utility function that allows for different levels of investor aversion to risk is employed. The marginal effects of changes in the expected returns of REITs versus stocks and the REIT-stock correlation on optimal portfolio weights for investors with different levels of risk

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aversion are developed mathematically. Understanding the impact of the mean-variance inputs on optimal asset allocations will lead to more informed decision making by investors.

Literature Review

The literature on the portfolio and diversification benefits of REITs and real estate as an asset class is rich and diverse. It is clear that real estate and REITs have a definite role in the formation of efficient portfolios. Fogler (1984), Firstinberg, Ross, and Zisler (1988), Ennis and Burik (1991), and Kallberg, Liu, and Greig (1996), among others, have suggested optimal allocations to real estate of approximately 10% to 20%. Giliberto (1992) used mean-variance analysis to show that if gross real estate returns range from 10% to 12%, then the optimal portfolio allocation to real estate should be from 5% to 10% of total assets. Webb, Curcio, and Rubens (1988) argued for much larger holdings in real estate, suggesting that a full two-thirds of the investor's portfolio should be allocated to real estate. Mueller, Pauley, and Morrill (1994) determined that REITs belong in efficient portfolios because they had returns similar to small company stocks, but with much less risk. Mueller and Mueller (2003) examined the effects of including both private real estate and REITs in a mixed-asset portfolio. They found that the asset categories had very low quarterly correlations and that both asset classes merit inclusion in the portfolio. Mueller and Mueller found unconstrained theoretical allocations to real estate in excess of 50%. A study update by Ibbotson (2003) showed that a 20% allocation to REITs increased expected portfolio returns at most risk levels by 50 basis points, while at the same time reducing portfolio risk. Goodman (2003) and Waggle and Johnson (2004) both found that REITs have a place in the portfolio of individual investors even when the family home is considered an asset.

Bajtelsmit and Worzala (1995) found that actual holdings of real estate were far below what would be considered optimal portfolio levels. In their sample of 159 pension funds, the actual allocations to real estate ranged from 0% to 17% with an average allocation of only 4.4%. This was also reinforced by the Benjamin, Sirmans, and Zietz (2001) survey paper that observed that although real estate is important for portfolio diversification, the actual amount of real estate held in mixed-asset portfolios was not as high as suggested by research. The study by Feldman (2003) also concluded that actual portfolio allocations to real estate have been far below optimal levels. This is particularly true in the case of REITs, which are supply-constrained in the short term. Feldman estimated that based on 2002 levels, the entire REIT market capitalization was equal to only about 3% of institutional holdings in financial assets. Survey papers by Seiler, Webb, and Myer (1999) and Benjamin, Sirmans, and Zietz (2001) explored the diversification literature as it relates to the real estate asset class and concluded that real estate does warrant inclusion in mixed-asset portfolios. They observed that researchers have not been able to identify and agree on an optimal level of real estate in a portfolio. Other surveys of real estate literature included Corgel, McIntosh, and Ott (1995), Norman, Sirmans, and Benjamin (1995), and Zietz, Sirmans, and Friday (2003).

Hudson-Wilson, Fabozzi, and Gordon (2003) emphasized that real estate is more than a mere diversifier. They recognized that real estate is a debt-equity hybrid and as such can provide additional benefits such as a reduction of overall portfolio risk, higher portfolio absolute returns, hedging unexpected inflation or deflation, generation of a wider investment universe, and the delivery of strong cash flows to the investor.

There are many challenges against the prospect of history repeating itself in the case of stocks and REITs. Ziering and McIntosh (1997) noted that the returns and correlations of stocks, bonds, REITs, and real estate varied considerably over subperiods from 1972 to 1995. Philips (1999), Arnott and Bernstein (2002), Hunt and Hoisington (2003), Ilmanen (2003), and Bernstein (1997a) all saw poor return prospects for stocks in the future. Wainscott (1990), Bernstein (1997b), and Dopfel (2003) all found the correlation matrix to be unstable over time. Mull and Soenen (1997) concluded that the diversification potential for REITs appears to be very time dependent.

Tactical asset allocation studies such as those by Bharati and Gupta (1992), Ilmanen (1997), and

Flavin and Wickens (2003) have attempted to improve on the historical inputs into mean-variance analysis. Along the same lines, Giliberto, Hamelink, Hoesli, and MacGregor (1999) used a QTARCH approach and find portfolio performance improvements from using conditional variances and covariances. Chandrashekaran (1999) estimated predicted means, variances, and covariances, which were then used to construct ex ante conditional mean-variance efficient portfolios and to study the ex post return characteristics of these portfolios. Chandrashekaran found that conditioning on lagged REIT returns allows for better predictions of volatilities and correlations of REITs with other asset classes. Anderson and Springer (2003) found performance persistence in REITs, which could continue into the second and third year. Chui, Titman, and Wei (2003) showed that expected returns of REITs were significantly different between the pre and the post-1990 subperiods, and in the post-1990 period, momentum was the dominant predictor of REIT returns.

Data

To examine the marginal effects of changes in mean-variance input items on optimal portfolio decisions, an initial set of assumptions is required. Historical return data for REITs, stocks, and bonds are employed to generate base level assumptions. This does not mean that these are the "predicted" levels. Rather, this allows for results that show portfolio decisions based on historical returns and the marginal effects of varying from those assumptions based on a predictive model or simply a belief that history will not repeat itself.

Monthly and annual total return data for equity REITs, large-company stocks, and long-term government bonds for the 1972 to 2002 period is the primary focus.[1] These asset categories will subsequently be referred to as simply REITs, stocks, and bonds to simplify the narrative. The return data for REITs is from the National Association of Real Estate Investment Trusts (NAREIT). The NAREIT Index, which has data back to 1972, tracks all publicly traded domestic REITs. Stock and bond total return data is obtained from the Ibbotson Associates SBBI 2003 Yearbook. While the Ibbotson data dates back to 1926, stock and bond data back is only presented here to 1972 to be consistent with the available NAREIT Index data.

Panel A of Exhibit 1 shows that for the 1972 to 2002 period, REITs outperformed stocks, managing a 13.5% average annual total return versus just 12.4% for stocks. The strong performance of REITs is even more impressive considering that they also maintained a lower level of volatility than stocks. The standard deviation of the REIT returns over the total period was 16.7% compared to 18.0% for stocks. The correlation matrices shown in Exhibit 1 are derived from comparisons of monthly total returns for the asset categories. Over the full period, REITs and stocks are strongly, positively correlated with a correlation coefficient of 0.53. In comparison, the REIT-bond correlation coefficient is 0.16.

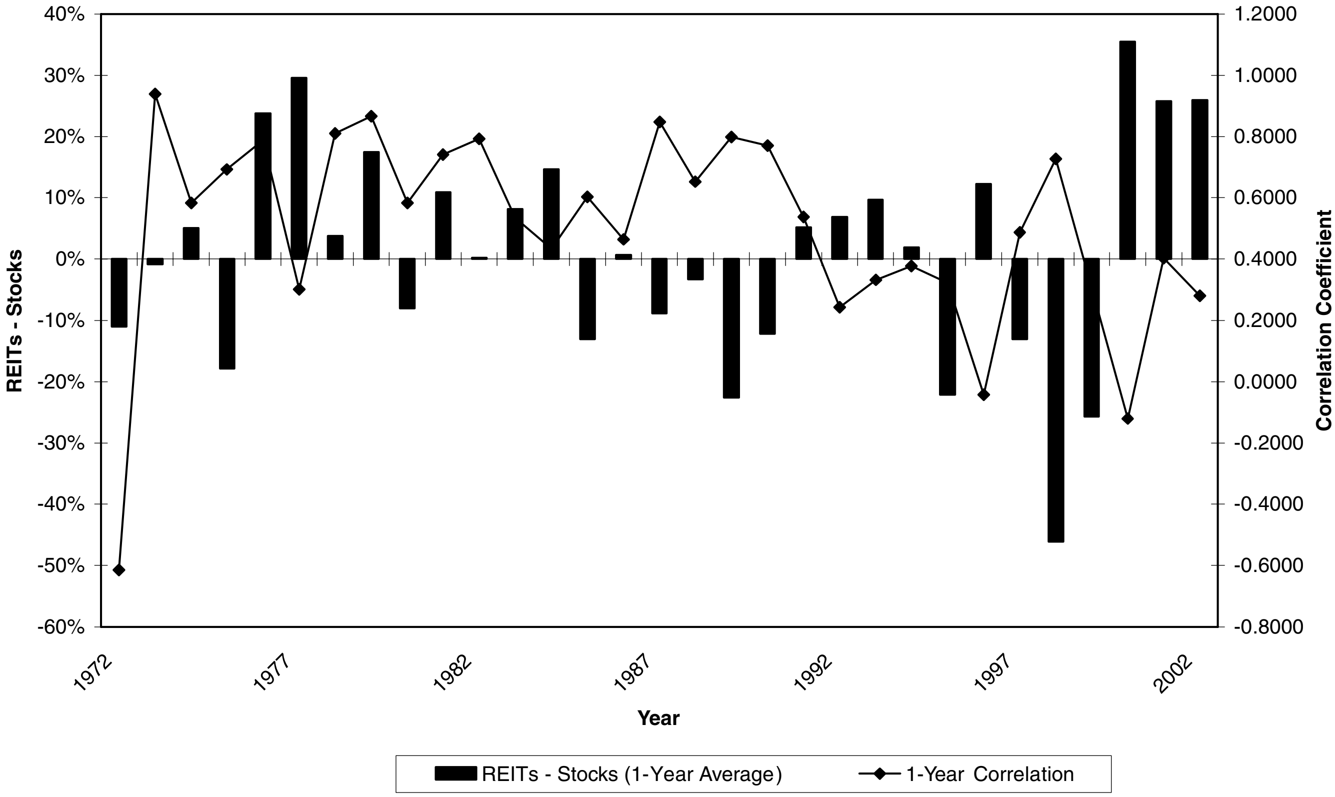
Exhibit 2 plots the annual excess return of REITs versus stocks and the twelve-month REIT-stock correlations for the period from 1972 to 2002. There were dramatic differences in the performance of REITs and stocks over this period. In 1998,

Exhibit 1
Key Statistics

	REITs	Stocks	Bonds
Panel A: 1972–2002			
Average	13.5%	12.4%	9.8%
Std. Dev.	16.7%	18.0%	12.2%
REITs	1.00		
Stocks	0.53	1.00	
Bonds	0.16	0.25	1.00
Panel B: 1972–1987			
Average	15.3%	11.9%	8.7%
Std. Dev.	17.8%	18.0%	13.3%
REITs	1.00		
Stocks	0.64	1.00	
Bonds	0.18	0.32	1.00
Panel C: 1988–2002			
Average	11.5%	13.0%	11.0%
Std. Dev.	15.8%	18.6%	11.2%
REITs	1.00		
Stocks	0.36	1.00	
Bonds	0.14	0.13	1.00

Notes: Means and standard deviations based on annual returns. Correlation matrices based on monthly returns.

Exhibit 2
REITs vs. Stocks: Excess Return and Correlation, 1972 to 2002 with 1-Year Non-Overlapping Periods

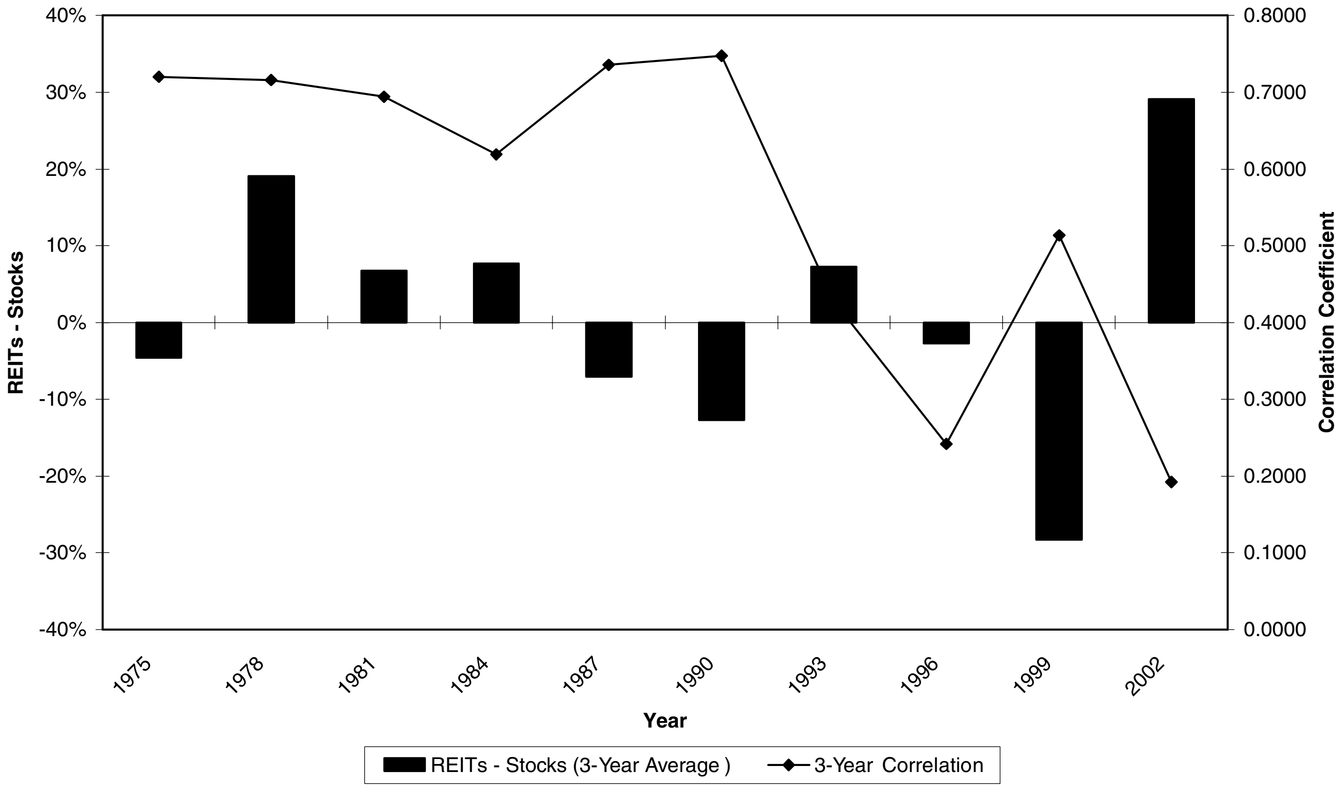


REITs demonstrated the worst relative performance, losing 17.5% compared to a 28.6% gain for stocks. The year 2000 proved to be a complete reversal of fortunes and the best relative performance for REITs, which returned 26.4% compared to a 9.1% loss for stocks. With the exception of the REIT index's first year and two recent periods, the REIT-stock correlation has been positive each year based on monthly returns.

Exhibits 3 and 4 both show the excess returns of REITs versus stocks and the REIT-stock correlation. Exhibit 3 reveals these figures for three-year, non-overlapping periods, and Exhibit 4 shows five-year, non-overlapping periods. The exhibits use 36 and 60 months, respectively, of trailing returns to calculate the correlation coefficients. The longer time periods in Exhibits 3 and 4 tend to smooth out the ups and downs seen in Exhibit 2. One obvious observation is that the correlation coefficient on the returns of REITs and stocks has declined over time. Panels B and C of Exhibit 1 exemplify the changes in REITs and the REIT-stock relationship that is seen over the 1972 to 2002 time period. Panel B of Exhibit 1 shows statistics for the 1972 to 1987 period, and Panel C of Exhibit 1 shows the 1988 to 2002 period. The REIT-stock correlation was 0.64 in the earlier period and declined to 0.36 in the later period.

The base-level data, which is employed as a starting point for determining marginal effects, is obtained from the statistics for the 1988 to 2002 period shown in Panel C of Exhibit 1. This latter half of the data encompasses more of the modern era of REITs. The Tax Reform Act of 1986 initiated significant changes that led to the subsequent growth of REITs. At the beginning of 1972, NAREIT's Equity REIT Index was based on just twelve REITs with a total market capitalization of only about $330 million. By the end of 1987, the number of publicly traded equity REITs had passed 50 with a total market capitalization approaching $5 billion. The popularity of REITs took off, and at the

Exhibit 3
REITs vs. Stocks: Excess Return and Correlation 1973 to 2002 with 3-Year Non-Overlapping Periods



end of 2002, there were 149 equity REITs with a total market capitalization of over $150 billion.[2]

The marginal effects calculations described below begin with assumed values based on the 1988 to 2002 period. The standard deviations of REITs, stocks, and bonds are assumed to be 15.8%, 18.6%, and 11.2%, respectively. The REIT-stock correlation is 0.36, and the REIT-bond correlation is 0.14. The stock-bond correlation is 0.13. These values remain constant in all of the analyses. What does vary in the analyses is the assumed differences in the returns between REITs and stocks and between stocks and bonds.

While this research work focuses on the REIT-stock relationship, the assumptions regarding the stock-bond relationship can have a significant impact on the findings. While the historical data for REITs dates back only to 1972, Ibbotson Associates has a much fuller set of stock and bond data dating back to 1926. The 2% premium that stocks earned relative to bonds in the 1988 to 2002 period was relatively small in historical terms. Over the 1926 to 2002 period, stocks outperformed bonds by about 6% per year. As noted earlier, there is disagreement among prognosticators about the future stock-bond risk premium. Accordingly, the analyses are conducted with stock-bond risk premiums of 2%, 4%, and 6%.

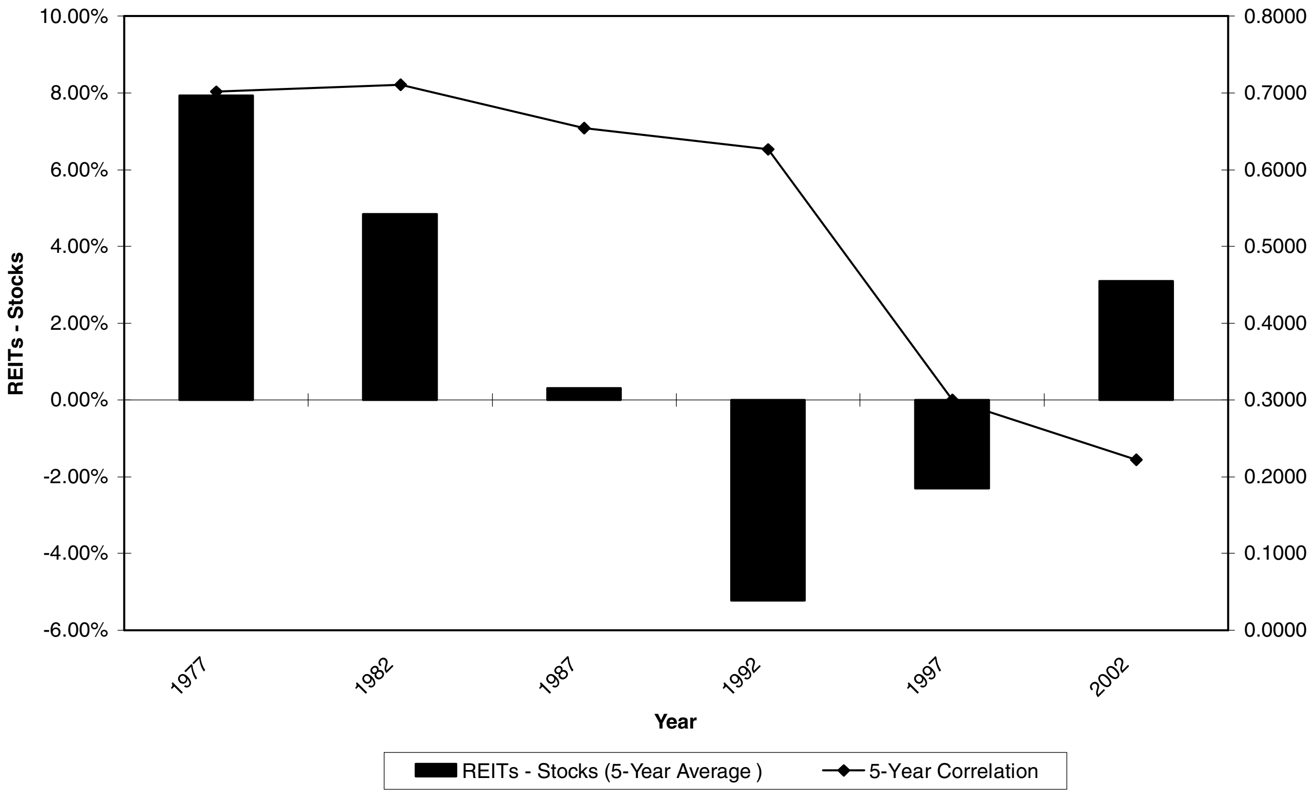
Mean-Variance and Utility

Assume that investors choose the portfolio weights that maximize utility U with the common function:[3]

$$U = r_p - \frac{1}{2} A \sigma_P^2. \tag{1}$$

While all risk-averse investors seek to avoid risk, different investors have different levels of risk aversion A. Low values of A are consistent with a

Exhibit 4
REITs vs. Stocks: Average Annual Excess Return and Correlation, 1972 to 2002 with 5-Year Non-Overlapping Periods



higher tolerance for risk, while higher values for A equate to higher degrees of risk aversion. Levels of A ranging from 1 to 10 are examined. The various levels of A actually plot the risk-return profiles of different investors along the efficient frontier. Investors with $A = 10$ have a high level of risk aversion and will choose portfolio combinations approaching the minimum variance portfolio. Investors with $A = 1$ will choose portfolio combinations that are much riskier and further to the right on the efficient frontier.

In the three-asset case with REITs, stocks, and bonds, the portfolio return is calculated as:

$$r_P = w_R r_R + w_S r_S + w_B r_B, \tag{2}$$

where w_R, w_S, and w_B are the portfolio weights of REITs, stocks, and bonds, respectively. These are the weights that investors adjust to maximize utility. The expected returns of REITs, stocks, and bonds are given by r_R, r_S, and r_B, respectively. The portfolio variance is calculated as:

$$\sigma_P^2 = w_S^2\sigma_S^2 + w_R^2\sigma_R^2 + w_B^2\sigma_B^2 + 2w_S w_R \rho_{SR}\sigma_S\sigma_R + 2w_S w_B \rho_{SB}\sigma_S\sigma_B + 2w_R w_B \rho_{RB}\sigma_R\sigma_B, \tag{3}$$

where ρ_{RS} is the REITs-stocks correlation, ρ_{SB} is the stocks-bonds correlation, and ρ_{RB} is the REITs-bonds correlation. The standard deviations of REITs, stocks, and bonds are given by σ_R, σ_S, and σ_B, respectively. Two constraints rule out short selling and assure portfolio completeness:

$$w_R, w_S, w_B \geq 0$$

and

$$w_R + w_S + w_B = 1. \tag{4}$$

The portfolios of more risk-averse investors are expected to include all three of the asset classes.

With the constraint against short selling, more aggressive investors (with lower values for A) will shift more of their assets toward REITs and stocks and eliminate their holdings in bonds. The most aggressive investment positions would include 100% REITs or stocks, depending on the return assumptions, eliminating the holdings of the other two asset classes. So the optimal portfolio solutions may include one, two, or three assets.

Optimal Portfolio Weights

The optimal portfolio weight in REITs w_R^* can be found by setting $\partial U/\partial w_R = 0$ and solving for w_R. Since the solutions to the three-asset case are quite involved, they are relegated to the Appendix. The optimal portfolio weight in REITs with just two assets is considered—REITs and stocks. With only these two assets, the portfolio return and standard deviation, which are substituted into Equation 1, are given by:

$$r_P = w_R r_R + (1 - w_R) r_S. \tag{5}$$

$$\sigma_P^2 = w_R^2 \sigma_R^2 + (1 - w_R)^2\, \sigma_S^2 + 2 w_R (1 - w_R) \rho_{RS} \sigma_R \sigma_S. \tag{6}$$

Since REITs and stocks comprise the entire portfolio, w_S is equal to $1 - w_R$. The optimal portfolio weight in REITs is then calculated as:

$$w_R^* = \frac{r_R - r_S + A(\sigma_S^2 - \rho_{RS}\sigma_R\sigma_S)}{A(\sigma_S^2 + \sigma_R^2 - 2\rho_{RS}\sigma_R\sigma_S)}, \tag{7}$$

and the optimal portfolio weight in stocks w_S^* is $1 - w_R^*$.

Marginal Effects

To find the marginal effect of changes in r_R on the optimal portfolio weight in REITs, $\partial w_R^*/\partial r_R$ is solved, which gives:

$$\frac{\partial w_R^*}{\partial r_R} = \frac{1}{A(\sigma_R^2 + \sigma_S^2 - 2\rho_{RS}\sigma_R\sigma_S)} > 0. \tag{8}$$

This demonstrates the effect of a change in the return of REITs on the optimal portfolio weight in REITs. Higher values for r_R lead to increased optimal weights for REITs in the portfolio. The degree of the increase in portfolio weight is dependent upon the variances and covariances of security returns and the risk aversion of the individual investor.

Finding the marginal effect of changes in ρ_{RS} on the optimal portfolio weight in REITs involves solving for $\partial w_R^*/\partial \rho_{RS}$, which gives:

$$\frac{\partial w_R^*}{\partial \rho_{RS}} = \frac{\sigma_R\sigma_S}{\sigma_R^2 + \sigma_S^2 - 2\rho_{RS}\sigma_S\sigma_R} \left[\frac{2[r_R - r_S + A(\sigma_S^2 - \rho_{RS}\sigma_R\sigma_S)]}{A(\sigma_S^2 + \sigma_R^2 - 2\rho_{RS}\sigma_R\sigma_S)} - 1 \right] \tag{9}$$

This reveals the impact that a change in the REIT-stock correlation will have on the optimal weight in REITs. The marginal effect is determined by the variances and covariances of security returns, the risk aversion parameter of the individual investor, the REIT risk premium ($r_R - r_S$), and the current REIT-stock correlation. The portfolio impacts on w_S^* are shown by $\partial w_S^*/\partial r_R$ and $\partial w_S^*/\partial \rho_{RS}$. In the two-asset case, $\partial w_S^*/\partial r_R$ is equal to $-\partial w_R^*/\partial r_R$, and $\partial w_S^*/\partial \rho_{RS}$ is equal to $-\partial w_R^*/\partial \rho_{RS}$.

To aid in interpretation, $\partial w_R^*/\partial r_R$ and $\partial w_S^*/\partial r_R$ are each divided by 100 to reveal the approximate changes in the respective portfolio weights of REITs and stocks per 1% change in r_R. $\partial w_R^*/\partial \rho_{RS}$ and $\partial w_S^*/\partial \rho_{RS}$ are both scaled by a factor of 10 to show the approximate changes in w_R^* and w_S^* per 0.1 change in ρ_{RS}. The optimal portfolio weights calculated with Equations 6 and A3 cannot be constrained to positive values. To overcome this shortcoming, the optimal portfolio weights were first determined using an optimization program with security weights constrained to non-negative values. The optimization output revealed whether the optimal portfolio combinations included one, two, or three assets. These findings determined which marginal effects equations were used.

Analysis

Using the formulas developed above and in the Appendix, the optimal portfolio combinations of REITs, stocks, and bonds are calculated, along with the marginal portfolio effects of changes in

Exhibit 5
Impact of Changes in REIT-Stock Correlation and Expected REIT Returns with $r_S - r_B$ = 2%

		A						
$r_R - r_S$		1	2	3	4	5	8	10
−1.5%	w_R^*	0.090	0.163	0.187	0.199	0.206	0.217	0.221
	w_S^*	0.676	0.406	0.315	0.270	0.243	0.202	0.189
	w_B^*	0.234	0.432	0.498	0.531	0.551	0.581	0.591
	$\partial w_R^*/\partial r_R$	0.403	0.201	0.134	0.101	0.081	0.050	0.040
	$\partial w_S^*/\partial r_R$	−0.173	−0.087	−0.058	−0.043	−0.035	−0.022	−0.017
	$\partial w_R^*/\partial \rho_{RS}$	−0.075	−0.040	−0.028	−0.022	−0.018	−0.013	−0.011
	$\partial w_S^*/\partial \rho_{RS}$	0.026	0.006	−0.001	−0.005	−0.007	−0.010	−0.011
−0.5%	w_R^*	0.493	0.364	0.321	0.299	0.287	0.267	0.261
	w_S^*	0.503	0.319	0.258	0.227	0.208	0.181	0.172
	w_B^*	0.004	0.317	0.422	0.474	0.505	0.552	0.568
	$\partial w_R^*/\partial r_R$	0.403	0.201	0.134	0.101	0.081	0.050	0.040
	$\partial w_S^*/\partial r_R$	−0.173	−0.087	−0.058	−0.043	−0.035	−0.022	−0.017
	$\partial w_R^*/\partial \rho_{RS}$	−0.034	−0.019	−0.014	−0.012	−0.010	−0.008	−0.007
	$\partial w_S^*/\partial \rho_{RS}$	−0.020	−0.017	−0.017	−0.016	−0.016	−0.015	−0.015
0.5%	w_R^*	**0.756**	0.565	0.455	0.400	0.367	0.318	0.301
	w_S^*	**0.244**	0.232	0.200	0.184	0.174	0.159	0.154
	w_B^*	**0.000**	0.202	0.345	0.416	0.459	0.523	0.545
	$\partial w_R^*/\partial r_R$	**0.260**	0.201	0.134	0.101	0.081	0.050	0.040
	$\partial w_S^*/\partial r_R$	**−0.260**	−0.087	−0.058	−0.043	−0.035	−0.022	−0.017
	$\partial w_R^*/\partial \rho_{RS}$	**0.039**	0.001	0.000	−0.001	−0.002	−0.003	−0.003
	$\partial w_S^*/\partial \rho_{RS}$	**−0.039**	−0.040	−0.032	−0.028	−0.025	−0.021	−0.020

Notes: Assumes σ_R, σ_S, σ_B, ρ_{RS}, ρ_{SB}, and ρ_{RB} are all at their historical levels based on the 1988 to 2002 time period as shown in Exhibit 1, Panel C.
$\partial w_R^*/\partial r_R$ and $\partial w_S^*/\partial r_R$ are divided by 100 to show approximate change per a 1% change in return.
$\partial w_R^*/\partial \rho_{RS}$ and $\partial w_S^*/\partial \rho_{RS}$ are divided by 10 to show the approximate change per a 0.1 change in correlation. Boldface type indicates the results of optimization results with just two assets rather than three. In these cases, unconstrained investors would have taken short bond positions.

the returns on REITs and the REIT-stock correlation. Exhibit 5 shows the results assuming that the stock-bond risk premium is only at the 2% level observed for the 1988 to 2002 period. The three sections of Exhibit 5 are predicated upon REITs underperforming stocks by 1.5% (as they did from 1988 to 2002), underperforming stocks by 0.5%, and outperforming stocks by 0.5%. The standard deviations and the correlation coefficients of REITs, stocks, and bonds are set at their 1988 to 2002 historical levels. Results are shown for A = 1, 2, 3, 4, 5, 8, and 10.

Looking at the formulas for w_R^* (Equation 7 for the two-asset case and Equation A5 for the three-asset case) reveals that the individual expected asset return levels are not what affects portfolio optimization decisions. It is actually the differences in the expected returns ($r_R - r_S$ and $r_S - r_B$) that are of interest. In the two-asset case, for instance, the optimal portfolio weights would be the same if REITs were expected to earn 11% compared to a stock return of 10% as if REITs were predicted to make 16% compared a stock return of 15%. Because of this, the results can be generalize by examining the REIT-stock risk premium or discount.

In the first panel of Exhibit 5 (stock-bond risk premium of 2% and REIT-stock risk premium of −1.5%), w_R^* is seen ranging from 9.0% for aggressive investors with A = 1 to a high of 22.1% for conservative investors with A = 10. At the same time, w_S^* declines from 67.6% for A = 1 to 18.9% for A = 10. This suggests that as risk aversion increases, investors should hold somewhat higher levels of REITs, but they should also offset the bulk of their stock positions with bonds.

In all cases, $\partial w_R^*/\partial r_R$ is much more significant than $\partial w_R^*/\partial \rho_{RS}$. A 1% swing in the expected REIT-stock risk premium will have a much more dramatic effect on the optimal portfolio composition, than a 0.1 movement in the expected REIT-stock correlation coefficient. With $A = 4$, for example, $\partial w_R^*/\partial r_R$ is 10.1% while $\partial w_R^*/\partial \rho_{RS}$ is just −2.2%. It would take approximately a 0.5 change in the expected correlation to have the same portfolio impact as a 1% change in asset return premiums. $\partial w_R^*/\partial r_R$ is heavily dependent on the degree of risk aversion of the individual investor. $\partial w_R^*/\partial \rho_{RS}$ is also dependent on the investor's level of risk aversion, but the differences are not as extreme. For $A = 1$, $\partial w_R^*/\partial \rho_{RS}$ is −7.5%, while it is just −1.1% for $A = 10$.

The assumptions behind Exhibits 6 and 7 mirror Exhibit 5, except that Exhibit 6 is based on a 4% stock-bond risk premium and Exhibit 7 is based on a 6% stock-bond risk premium. These are both considerably larger than the 2% average stock-bond risk premium observed over the 1988 to 2002 period, which was the basis of Exhibit 5. Comparisons of Exhibits 5, 6, and 7 reveals that lower bond return forecasts lead to considerably larger optimal portfolio allocations to REITs across all levels of REIT-stock risk premiums and investor risk tolerance levels, as shown by the variable A.

Looking down the columns in Exhibits 5, 6, and 7 reveals that $\partial w_R^*/\partial r_R$ and $\partial w_S^*/\partial r_R$ are constant in cases except when comparing two-asset portfolio solutions to three-asset solutions. Examination of Equations 7 (two-asset case) and A6 (three-asset case) shows that the level of the returns do not enter into the solutions. The marginal effects due to changes in returns are affected by the variances and covariances of the asset returns and the level of risk aversion of the individual investor, but not by the current return levels. This is also evident

Exhibit 6

Impact of Changes in REIT-Stock Correlation and Expected REIT Returns with $r_S - r_B = 4\%$

		A						
$r_R - r_S$		1	2	3	4	5	8	10
−1.5%	w_R^*	**0.235**	**0.392**	0.340	0.314	0.298	0.274	0.266
	w_S^*	**0.765**	**0.547**	0.409	0.341	0.299	0.238	0.217
	w_B^*	**0.000**	**0.061**	0.251	0.346	0.403	0.488	0.517
	$\partial w_R^*/\partial r_R$	**0.260**	**0.201**	0.134	0.101	0.081	0.050	0.040
	$\partial w_S^*/\partial r_R$	**−0.260**	**−0.087**	−0.058	−0.043	−0.035	−0.022	−0.017
	$\partial w_R^*/\partial \rho_{RS}$	**−0.041**	**−0.045**	−0.031	−0.024	−0.020	−0.014	−0.012
	$\partial w_S^*/\partial \rho_{RS}$	**0.041**	**−0.008**	−0.011	−0.012	−0.012	−0.013	−0.014
−0.5%	w_R^*	**0.495**	**0.560**	0.474	0.414	0.378	0.325	0.307
	w_S^*	**0.505**	**0.440**	0.352	0.297	0.265	0.216	0.200
	w_B^*	**0.000**	**0.000**	0.175	0.288	0.357	0.459	0.494
	$\partial w_R^*/\partial r_R$	**0.260**	**0.130**	0.134	0.101	0.081	0.050	0.040
	$\partial w_S^*/\partial r_R$	**−0.260**	**−0.130**	−0.058	−0.043	−0.035	−0.022	−0.017
	$\partial w_R^*/\partial \rho_{RS}$	**−0.001**	**0.009**	−0.017	−0.014	−0.012	−0.009	−0.008
	$\partial w_S^*/\partial \rho_{RS}$	**0.001**	**−0.009**	−0.026	−0.023	−0.021	−0.019	−0.018
0.5%	w_R^*	**0.756**	**0.691**	0.608	0.515	0.459	0.375	0.347
	w_S^*	**0.244**	**0.309**	0.294	0.254	0.230	0.194	0.182
	w_B^*	**0.000**	**0.000**	0.098	0.231	0.311	0.431	0.471
	$\partial w_R^*/\partial r_R$	**0.260**	**0.130**	0.134	0.101	0.081	0.050	0.040
	$\partial w_S^*/\partial r_R$	**−0.260**	**−0.130**	−0.058	−0.043	−0.035	−0.022	−0.017
	$\partial w_R^*/\partial \rho_{RS}$	**0.039**	**0.029**	−0.004	−0.004	−0.004	−0.004	−0.004
	$\partial w_S^*/\partial \rho_{RS}$	**−0.039**	**−0.029**	−0.041	−0.035	−0.031	−0.025	−0.023

Notes: Assumes σ_R, σ_S, σ_B, ρ_{RS}, ρ_{SB}, and ρ_{RB} are all at their historical levels based on the 1988 to 2002 time period as shown in Exhibit 1, Panel C.

$\partial w_R^*/\partial r_R$ and $\partial w_S^*/\partial r_R$ are divided by 100 to show approximate change per a 1% change in return.

$\partial w_R^*/\partial \rho_{RS}$ and $\partial w_S^*/\partial \rho_{RS}$ are divided by 10 to show the approximate change per a 0.1 change in correlation.

Boldface type indicates the results of optimization results with just two assets rather than three. In these cases, unconstrained investors would have taken short bond positions.

Exhibit 7

Impact of Changes in REIT-Stock Correlation and Expected REIT Returns with $r_S - r_B = 6\%$

		A						
$r_R - r_S$		1	2	3	4	5	8	10
−1.5%	w_R^*	**0.235**	**0.430**	0.493	0.428	0.390	0.332	0.312
	w_S^*	**0.765**	**0.570**	0.503	0.411	0.356	0.273	0.245
	w_B^*	**0.000**	**0.000**	0.004	0.161	0.255	0.395	0.442
	$\partial w_R^*/\partial r_R$	**0.260**	**0.130**	0.134	0.101	0.081	0.050	0.040
	$\partial w_S^*/\partial r_R$	**−0.260**	**−0.130**	−0.058	−0.043	−0.035	−0.022	−0.017
	$\partial w_R^*/\partial \rho_{RS}$	**−0.041**	**−0.011**	−0.034	−0.027	−0.022	−0.015	−0.013
	$\partial w_S^*/\partial \rho_{RS}$	**0.041**	**0.011**	−0.020	−0.019	−0.018	−0.017	−0.016
−0.5%	w_R^*	**0.495**	**0.560**	0.582	0.529	0.470	0.382	0.353
	w_S^*	**0.505**	**0.440**	0.418	0.368	0.321	0.251	0.228
	w_B^*	**0.000**	**0.000**	0.000	0.103	0.209	0.367	0.419
	$\partial w_R^*/\partial r_R$	**0.260**	**0.130**	0.087	0.101	0.081	0.050	0.040
	$\partial w_S^*/\partial r_R$	**−0.260**	**−0.130**	−0.087	−0.043	−0.035	−0.022	−0.017
	$\partial w_R^*/\partial \rho_{RS}$	**−0.001**	**0.009**	0.013	−0.017	−0.014	−0.010	−0.009
	$\partial w_S^*/\partial \rho_{RS}$	**0.001**	**−0.009**	−0.013	−0.030	−0.027	−0.022	−0.021
0.5%	w_R^*	**0.756**	**0.691**	0.669	0.630	0.551	0.432	0.393
	w_S^*	**0.244**	**0.309**	0.331	0.325	0.287	0.230	0.211
	w_B^*	**0.000**	**0.000**	0.000	0.046	0.163	0.338	0.397
	$\partial w_R^*/\partial r_R$	**0.260**	**0.130**	0.087	0.101	0.081	0.050	0.040
	$\partial w_S^*/\partial r_R$	**−0.260**	**−0.130**	−0.087	−0.043	−0.035	−0.022	−0.017
	$\partial w_R^*/\partial \rho_{RS}$	**0.039**	**0.029**	0.026	−0.006	−0.006	−0.005	−0.005
	$\partial w_S^*/\partial \rho_{RS}$	**−0.039**	**−0.029**	−0.026	−0.042	−0.036	−0.028	−0.026

Notes: Assumes σ_R, σ_S, σ_B, ρ_{RS}, ρ_{SB}, and ρ_{RB} are all at their historical levels based on the 1988 to 2002 time period as shown in Exhibit 1, Panel C.

$\partial w_R^*/\partial r_R$ and $\partial w_S^*/\partial r_R$ are divided by 100 to show approximate change per a 1% change in return.

$\partial w_R^*/\partial \rho_{RS}$ and $\partial w_S^*/\partial \rho_{RS}$ are divided by 10 to show the approximate change per a 0.1 change in correlation. Boldface type indicates the results of optimization results with just two assets rather than three. In these cases, unconstrained investors would have taken short bond positions.

when comparing the same levels of A across Exhibits 5, 6, and 7. Changes in the perceived stock-bond risk premium will affect w_R^*, w_S^*, and w_B^*, but not $\partial w_R^*/\partial r_R$ and $\partial w_S^*/\partial r_R$.

$\partial w_R^*/\partial \rho_{RS}$ and $\partial w_S^*/\partial \rho_{RS}$ do vary as expectations about returns vary. As seen in Equation A7, both $r_R - r_S$ and $r_S - r_B$ enter into the optimal solution. In Exhibit 5, with $A = 4$, $\partial w_R^*/\partial \rho_{RS}$ is -2.8% when $r_R - r_S = -1.5\%$, and it is just -0.10% when $r_R - r_S = 0.5\%$. Changes in $r_S - r_B$ also affect $\partial w_R^*/\partial \rho_{RS}$ and $\partial w_S^*/\partial \rho_{RS}$, but looking across Exhibits 5, 6, and 7 reveals that the changes are minimal.

Conclusion

Real estate investment trusts are a common fixture in mixed-asset portfolios of individual investors, but investors must be concerned with determining the appropriate level of REITs that they should hold in their portfolios. Investors can maximize their expected utility by forming mean-variance efficient portfolio combinations of REITs and other financial assets. Mean-variance efficient portfolios allow investors to maximize expected return for given levels of risk or minimize risk for given return levels. With mean-variance analysis, investors can identify the optimal portfolio weights of REITs and other assets.

Finding mean-variance efficient portfolio combinations requires certain assumptions about the expected returns, standard deviations, and correlations of assets. If the estimates that go into the process are off, then the portfolio selections based on them will be off as well and sub-optimal. Of particular concern for REIT investors is the relationship between REITs and stocks. The NAREIT Index dates back only about thirty years to 1972. Many researchers argue, however, that meaningful

REIT data dates back far less than thirty years, suggesting that the modern era of REITs began after tax reforms in 1986 or in the 1990s when the dollar values of REITs began to reach critical mass. Regardless, the limited history of REITs makes predicting the future based on the past somewhat problematic.

It is obvious that changes in estimates or errors in mean-variance inputs will have an impact on portfolio allocations, but to what extent? This work was centered on modeling the marginal effects of changes in the REIT-stock risk premium and the REIT-stock correlation. The findings indicate that the expected return of REITs relative to that of stocks is a much more important factor than the REIT-stock correlation in making portfolio decisions. A 1% change in the forecast return for REITs, holding the stock return fixed, has a dramatic impact in optimal portfolio allocations for investors of all risk levels. For investors with a moderate level of risk aversion, $A = 3$, a 1% decrease in the expected return on REITs would reduce the optimal holding of REITs by over 13%. The portfolio impacts due to changes in REIT returns are more pronounced for aggressive investors and less so for more conservative investors. A 0.1 change in the REIT-stock correlation, on the other hand, has minimal portfolio impact for investors across most risk levels. For most investors, in most of the scenarios considered here, a 0.1 shift in the REIT-stock correlation would change the optimal weight in REITs by less than 2%.

For many investors, the marginal effects calculated in this research reveal that their actual, as compared to theoretical optimal, allocations will not be affected at all. Given that mean-variance efficient portfolio results can swing dramatically, many investors prefer constrained optimization. Ibbotson (2003), for instance, assumed either 10% or 20% allocations to REITs. Except for the most aggressive investors ($A = 1$ to 3), employing the most pessimistic forecast of REIT returns in the analysis ($r_R - r_S = -1.5\%$), the marginal effects of a 1% reduction in REIT returns or a 0.1 increase in the REIT-stock correlation are not sufficient to affect a 10% maximum portfolio weight constraint on REITs. Likewise, for the majority of scenarios, a 20% constraint would be unaffected by the noted swings in REIT returns and the REIT-stock correlation.

Appendix

The derivations of optimal portfolio weights and marginal effects terms for the three-asset portfolio (REITs, stocks, and bonds) are quite straight forward, but the results are somewhat involved, as shown below. Substituting $(1 - w_R - w_B)$ for w_S into Equations 2 and 3 puts the problem in terms of just two unknowns:

$$r_P = w_R r_R + (1 - w_R - w_B) r_S + w_B r_B. \quad \text{(A1)}$$

$$\begin{aligned}\sigma_P^2 = {} & w_R^2\sigma_R^2 + (1 - w_R - w_B)^2\,\sigma_S^2 + w_B^2\sigma_B^2 \\ & + 2w_R(1 - w_R - w_B)\sigma_{RS} + 2w_R w_B \sigma_{RB} \\ & + 2(1 - w_R - w_B) w_B \sigma_{SB}. \quad \text{(A2)}\end{aligned}$$

The covariances σ_{IJ} of the asset pairs are determined by the individual variability of the assets and their correlations with each other: $\sigma_{IJ} = \rho_{IJ}\sigma_I\sigma_J$.

Setting $\partial U/\partial w_R = 0$ and solving for w_R results in:

$$w_R^* = \frac{r_R - r_S + A(\sigma_S^2 - \sigma_{RS}) - w_B A(\sigma_S^2 - \sigma_{RS} - \sigma_{RB} - \sigma_{SB})}{A(\sigma_R^2 + \sigma_S^2 - 2\sigma_{RS})}. \quad \text{(A3)}$$

In this solution, the optimal portfolio weight in REITs w_R^* is a function of w_B, which is also unknown. Setting $\partial U/\partial w_B = 0$ and solving for w_B results in:

$$w_B^* = \frac{r_B - r_S + A(\sigma_S^2 - \sigma_{SB}) - w_R A(\sigma_S^2 - \sigma_{RS} - \sigma_{RB} - \sigma_{SB})}{A(\sigma_B^2 + \sigma_S^2 - 2\sigma_{SB})}. \quad \text{(A4)}$$

Substituting w_B^* of Equation A4 in for w_B in Equation A3, after some rearranging, leads to the following:

$$w_R^* = \frac{[r_R - r_S + A(\sigma_S^2 - \sigma_{RS})]Y - [r_B - r_S + A(\sigma_S^2 - \sigma_{SB})]Z}{A(XY - Z^2)}, \quad \text{(A5)}$$

where:

$$X = (\sigma_R^2 + \sigma_S^2 - 2\sigma_{RS}).$$

$$Y = (\sigma_S^2 + \sigma_B^2 - 2\sigma_{SB}).$$

$$Z = (\sigma_S^2 - \sigma_{RS} + \sigma_{RB} - \sigma_{SB}).$$

Solving for $\partial w_R^*/\partial r_R$ shows the impact of a change in r_R on the optimal portfolio weight of REITs and is given by:

$$\frac{\partial w_R^*}{\partial r_R} = \frac{Y}{A(XY - Z^2)} \tag{A6}$$

To find the marginal impact of changes in ρ_{RS} on the optimal portfolio weight in REITs, solve for $\partial w_R^*/\partial \rho_{RS}$, which gives:

$$\frac{\partial w_R^*}{\partial \rho_{RS}} = \frac{\begin{array}{c}2\sigma_R\sigma_S Y(Y - Z)[r_R - r_S + A(\sigma_S^2 - \sigma_{RS})] \\ - A\sigma_R\sigma_S Y(XY - Z^2)\end{array}}{A(XY - Z^2)^2} + \frac{\begin{array}{c}\sigma_R\sigma_S[(XY - Z^2) + 2\sigma_R\sigma_S Z(Z - Y)] \\ [r_B - r_S + A(\sigma_S^2 - \sigma_{SB})]\end{array}}{A(XY - Z^2)^2}. \tag{A7}$$

Endnotes

1. While there are also mortgage REITs and hybrid equity-mortgage REITs, this work focuses only on the pure ownership positions. Equity REITs are currently by far the largest category, comprising approximately 90% of the market value of the publicly traded REITs. See www.NAREIT.com.
2. See www.NAREIT.com for more details on the composition of the REIT index.
3. See, for example, Bodie, Kane, and Marcus (2005).

References

Anderson, R. I. and T. M. Springer. REIT Selection and Portfolio Construction: Using Operating Efficiency as an Indicator of Performance. *Journal of Real Estate Portfolio Management*, 2003, 9:1, 17–28.

Arnott, R. D. and P. L. Bernstein. What Risk Premium is Normal?, *Financial Analysts Journal*, 2002, 58:2, 64–85.

Bajtelsmit, V. L. and Elaine M. Worzala. Real Estate Allocations in Pension Fund Portfolios. *Journal of Real Estate Portfolio Management*, 1995, 1, 25–38.

Benjamin, J. D., G. S. Sirmans, and E. N. Zietz. Returns and Risk on Real Estate and Other Investments: More Evidence. *Journal of Real Estate Portfolio Management*, 2001, 7, 183–214.

Bernstein, P. L. What Rate of Return Can You Reasonably Expect . . . Or What Can the Long Run Tell Us About the Short Run? *Financial Analysts Journal*, 1997a, 53:2, 20–8.

——., How To Own Bonds and Enjoy It. *Journal of Portfolio Management*, 1997b, 23:4, 1.

Bharati, R. and M. Gupta. Asset Allocation and Predictability of Real Estate Returns. *Journal of Real Estate Research*, 1992, 7:4, 469–84.

Bodie, Z., A. Kane, and A. Marcus. *Essentials of Investments*. Fifth edition. McGraw-Hill/Irwin Series in Finance, Insurance, and Real Estate. Irwin/McGraw-Hill, 2003.

Chandrashekaran, V. Time-Series Properties and Diversification Benefits of REIT Returns. *Journal of Real Estate Research*, 1999, 17, 91–112.

Chui, A. C. W., S. Titman, and K. C. J. Wei. The Cross Section of Expected REIT Returns. *Real Estate Economics*, 2003, 31:3, 451–79.

Corgel, J. B., W. McIntosh, and S. H. Ott. Real Estate Investment Trusts: A Review of the Financial Economics Literature. *Journal of Real Estate Literature*, 1995, 3, 13–43.

Dopfel, F. E. Asset Allocation in a Lower Stock-Bond Correlation Environment. *Journal of Portfolio Management*, 2003, 30: 1, 25–38.

Ennis, R. M. and P. Burik. Pension Fund Real Estate Investment Under a Simple Equilibrium Pricing Model. *Financial Analysts Journal*, 1991, May–June, 20–30.

Feldman, B. E. Investment Policy for Securitized and Direct Real Estate. *Journal of Portfolio Management*, 2003, September, 112–21.

Firstinberg, P., S. Ross, and R. Zisler. Real Estate: The Whole Story. *Journal of Portfolio Management*, 1988, 14:3, 22–34.

Flavin, T. J. and M. R. Wickens. Macroeconomic Influences on Optimal Asset Allocation. *Review of Financial Economics*, 2003, 12:2, 207–31.

Fogler, H. R., 20% in Real Estate: Can Theory Justify It?, *Journal of Portfolio Management*, 1984, Winter, 6–13.

Giliberto, S. M. The Allocation of Real Estate to Future Mixed-Asset Institutional Portfolios. *Journal of Real Estate Research*, 1992, 7:4, 423–32

Giliberto, M., F. Hamelink, M. Hoesli, and B. MacGregor. Optimal Diversification within Mixed-Asset Portfolios Using a Conditional Heteroscedasticity Approach: Evidence From the U.S. and the U.K. *Journal of Real Estate Portfolio Management*, 1999, 5:1, 31–45.

Goodman, J. Homeownership and Investment in Real Estate Stocks. *Journal of Real Estate Portfolio Management*, 2003, 9: 2, 93–105.

Hudson-Wilson, S., F. J. Fabozzi, and J. N. Gordon. Why Real Estate?, *Journal of Portfolio Management*, 1995, 20:3, 26–8.

Hunt, L. H. and D. M. Hoisington. Estimating the Stock/Bond Risk Premium. *Journal of Portfolio Management*, 2003, 29:2, 28–34.

Ibbotson Associates, New Analysis Validates Benefits of Including Real Estate as a Core Portfolio Holding. Press release dated September 17, 2003. Available at www.NAREIT.com.

Ilmanen, A. Forecasting U.S. Bond Returns. *Journal of Fixed Income*, 1997, 7:1, 22–37.

——., Expected Returns on Stocks and Bonds. *Journal of Portfolio Management*, 2003, 29:2, 7–27.

Kallberg, J. G., C. H. Liu, and W. D. Greig. The Role of Real Estate in the Portfolio Allocation Process. *Real Estate Economics*, 1996, 24, 359–77.

Mueller, A. G. and G. R. Mueller. Public and Private Real Estate in a Mixed-Asset Portfolio. *Journal of Real Estate Portfolio Management*, 2003, 9:3, 193–203.

Mueller, G. R., K. Pauley, and W. A. Morrell. Should REITs be Included in a Mixed-Asset Portfolio? *Real Estate Finance*, 1994, 11:1, 23–8.

Mull, S. and L. Soenen. U.S. REITs as an Asset Class In International Investment Portfolio. *Financial Analysts Journal*, 1997, 53, 55–61.

Norman, E. J., G. S. Sirmans, and J. D. Benjamin. The Historical Environment of Real Estate Returns. *Journal of Real Estate Portfolio Management*, 1995, 1:1, 1–24.

Philips, T. K. Why do Valuation Ratios Forecast Long-Run Equity Returns? *Journal of Portfolio Management*, 1999, 25:3, 39–44.

Seiler, M. J., J. R. Webb, and F. C. N. Myer. Are REITs Real Estate? *Journal of Real Estate Portfolio Management*, 1999, 5:2, 171–81.

Waggle, D. and D. T. Johnson. Home Ownership and the Decision to Invest in REITs. *Journal of Real Estate Portfolio Management*, 2004, 10:2, 129–44.

Wainscott, C. B. The Stock-Bond Correlation and Its Implications for Asset Allocation. *Financial Analysts Journal*, 1990, 46:4, 55–60.

Webb, J. R., R. J. Curcio, and J. H. Rubens. Diversification Gains from including Real Estate in Mixed-Asset Portfolios. *Decision Sciences*, 1988, 19:3, 434–52.

Ziering, B. and W. McIntosh. Revisiting the Case for Including Core Real Estate in a Mixed-Asset Portfolio. *Real Estate Finance*, 1997, 14:4, 14–22.

Zietz, E. N., G. S. Sirmans, and H.S. Friday. The Environment and Performance of Real Estate Investment Trusts. *Journal of Real Estate Portfolio Management*, 2003, 9:2, 127–65.